Differential and Integral Equations, Volume 5, Number 1, January 1992, pp. 55–72.

POSITIVE PERIODIC SOLUTIONS OF LOTKA-VOLTERRA REACTION-DIFFUSION SYSTEMS

JULIÁN LÓPEZ-GÓMEZ
Departamento de Matemática Aplicada
Universidad Complutense de Madrid, 28040 Madrid, Spain

(Submitted by: Peter Hess)

Abstract. A general existence result of positive solutions in both components for the Lotka-Volterra R-D systems with time periodic and spatially dependent coefficients is given. Predator-prey, competing and cooperating interactions are included in the abstract framework. The fixed point index is used to prove the main result. Optimal coexistence results for the associated elliptic model subject to homogeneous Dirichlet boundary conditions and allowing the various coefficients in the model to be spatial dependent are also obtained.

1. Introduction. In this paper we study the existence of strictly positive solutions of the following problem

$$
\begin{aligned}
\partial_t u - d_1(t)\Delta u &= \lambda(x,t)u - a(x,t)u^2 - b(x,t)uv, \quad \text{in } \Omega \times [0,\infty),\\
\partial_t v - d_2(t)\Delta v &= \mu(x,t)v - c(x,t)uv - d(x,t)v^2, \quad \text{in } \Omega \times [0,\infty),\\
u(x,t) &= v(x,t) = 0, \quad \text{for } x \in \partial\Omega, \quad t > 0,\\
u(x,0) &= u(x,T), \quad v(x,0) = v(x,T), \quad x \in \Omega,
\end{aligned}
\tag{1.1}
$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open set with smooth boundary $\partial\Omega$, $d_1(t)$, $d_2(t)$ are smooth strictly positive T-periodic functions and λ, μ, a, b, c, d are smooth functions on $\bar{\Omega} \times \mathbb{R}$ which are assumed to be T-periodic in t.

Some assumptions concerning the sign of the various coefficients in the model will be necessary. Concretely, $a(x,t)$ and $d(x,t)$ will be positive in $\bar{\Omega} \times \mathbb{R}$ and $b(x,t)$, $c(x,t)$ will have a constant sign in $\bar{\Omega} \times \mathbb{R}$ each separately. Moreover, if $b > 0$ and $c > 0$, we shall assume $\lambda > 0$ and $\mu > 0$; if $b > 0$ and $c < 0$, we shall assume $\lambda > 0$; if $b < 0$ and $c > 0$, we shall consider $\mu > 0$; finally, if $b < 0$ and $c < 0$, we shall assume $a_L d_L - b_L c_L > 0$, where given a continuous function $f : \bar{\Omega} \times [0,T] \to \mathbb{R}$ we have denoted $f_L \equiv \min\{f(x,t) : (x,t) \in \bar{\Omega} \times [0,T]\}$. These constraints on the sign of the coefficients are needed in order to have a priori bounds for the positive solutions in both components of the problem (1.1).

In references [3], [5], [6] and [7], previous work can be found concerning the stated problem. In [3] a necessary and sufficient condition for existence of positive solutions in both components in the case $\lambda > 0$, $\mu < 0$, $b > 0$, $c < 0$ was given. In [7], working under nonflux boundary conditions, some sufficient conditions assuming that $b > 0$

Received for publication in revised form November 1990.
AMS Subject Classifications: 35B10.