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DESTABILIZATION BY OUTPUT FEEDBACK

D. HINRICHSEN
Institut für Dynamische Systeme, Universität Bremen, D-2800 Bremen 33, F.R.G.

A.J. PRITCHARD
Control Theory Centre, University of Warwick, Coventry CV4 7AL, UK

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Abstract. In this paper, we investigate the robustness of stability of linear systems $\dot{x} = Ax$ respectively $x(t+1) = Ax(t)$ under complex or real (i) time-varying linear, (ii) nonlinear, (iii) time-varying nonlinear, or (iv) dynamic parameter perturbations of the output feedback type. The class of dynamic perturbations is sufficiently wide to account for all kinds of neglected dynamics with finite gain encountered in applications, including perturbation operators with unbounded memory. For each of the above perturbation classes, the *stability radius* of the given system is defined to be the norm of the smallest destabilizing perturbation. If complex perturbations are considered, the four corresponding stability radii turn out to be identical, whereas for real perturbations the stability radii depend on the specific perturbation class. These results are established for both discrete and continuous time systems. In the differentiable case, a multivariable extension of the Aizerman conjecture is discussed in detail for both real and complex nonlinearities.

1. Introduction. In control engineering and other application areas, a linear dynamic model of the form

$$\text{(a)}\quad \dot{x}(t) = Ax(t),\ \ t \in \mathbb{R}_+, \quad \text{or} \quad \text{(b)}\quad x(t+1) = Ax(t),\ t \in \mathbb{N}, \tag{1}$$

($A \in \mathbb{K}^{n\times n}$, $\mathbb{K} = \mathbb{R}, \mathbb{C}$) will usually contain uncertain parameters. It may have been obtained by simplification of a more "realistic" model, e.g., by linearization of a nonlinear system around an equilibrium point, by neglecting time variations or time delays, by order reductions or by finite dimensional approximation of an infinite-dimensional system. In all these cases, if the nominal model (1) is (*asymptotically*) *stable*, i.e., $\sigma(A) \subset \mathbb{C}_- = \{s \in \mathbb{C} : \operatorname{Re} s < 0\}$, respectively $\sigma(A) \subset \mathbb{C}_1 = \{s \in \mathbb{C} : |s| < 1\}$, it is important to ensure that the system remains stable for the whole range of possible parameter values or for the more "realistic" model which it approximates. Viewing this "realistic" model as a perturbation of the nominal one, it is natural to ask what is the largest bound $r > 0$ such that stability is preserved for all perturbations of norm strictly less than r in a given normed perturbation space $\mathcal{P}$. Clearly, this *problem of robust stability* can equivalently be stated as a minimum norm *destabilization problem.*

Whilst there is a wealth of qualitative information available concerning the behaviour of parametrized systems under small parameter perturbations, the fields of

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