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ON NONLINEAR VOLTERRA EQUATIONS IN HILBERT SPACES*

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Abstract. It will be shown that Volterra equations of form $A(t)u(t) + a * B(t)u(t) + 1 * Cu(t) \ni f(t)$ have a solution. The operators $A(t)$ and $B(t)$ are subdifferentials in a real Banach space which is embedded compactly and densely to a real Hilbert space, in which $A(t)$ is or is not compact. The operators $C(t)$ are Lipschitzian and compact. Finally, some remarks on the uniqueness of the solution are given.

1. Introduction.

We consider the following nonlinear Volterra equation

$$A(t)u(t) + \int_0^t a(t-s)B(s)u(s)\,ds + \int_0^t C(s)u(s)\,ds \ni f(t), \quad 0 < t < \infty, \tag{1.1}$$

where the values of $u(\cdot)$ belongs to a real reflexive Banach space, $a(\cdot)$ is a scalar function, $A(t)$ and $B(t)$ are subdifferentials, $A(t)$ is or is not compact and $C(t)$ is a Lipschitzian compact operator. If $a \equiv 1$, then the problems (1.1) and (1.2),

$$\frac{d}{dt}A(t)u(t) + B(t)u(t) + C(t)u(t) \ni f'(t), \quad 0 < t < \infty, \tag{1.2a}$$

$$A(0)u(0) \ni f(0), \tag{1.2b}$$

are equivalent. The classical theory for the Cauchy equation ([3], [7]) can not be applied to the problem (1.2), but there are works on (1.2) in the case where A and B do not depend on time and $C \equiv 0$ ([5] and [10]); in the case where A depend on time and C is not zero, ([6]); and in the case where even B depends on time, ([12]). V. Barbu has studied the problem (1.1) in the case where A and B do not depend on time and $C \equiv 0$.

We shall apply the methods of Barbu with some modification. The problem (1.1) is regularized by the term $\epsilon G(t)u(t) + A(t)u(t)$ instead of $A(t)u(t)$; $G(t)$ is some coersive maximal monotone operator. The regularized problem is regularized more by the Yosida approximate $B_\lambda(t)$ instead of $B(t)$. A corollary of the fixed point theorem implies that this problem has a solution. By taking the limits $\lambda \to 0+$

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