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ON THE SOLVABILITY OF n-th ORDER BOUNDARY VALUE PROBLEMS BETWEEN TWO EIGENVALUES*

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Abstract. We are concerned with the solvability of the nonlinear ordinary differential equation $Dy + f(x,y) = h(x)$, subjected to the linear boundary condition $B(y,\dots,y^{(n-1)}) = r$, where D is a $n-$th order linear differential operator. As well-known, existence results for problems of this kind are usually obtained under suitable conditions of noninterference of the ratio $f(x,y)/y$ (for $|y|$ large) with the spectrum of the differential operator $-D$ with associated homogeneous boundary condition. Sharper results are obtained in the self-adjoint case. In the present work we are mainly interested in the non self-adjoint case and show by a class of counterexamples that the usual assumptions of "nonresonance between two eigenvalues" are not sufficient, in general, to guarantee the solvability of the considered BVPs. Existence results are obtained as well.

1. Introduction, setting of the problem and general existence results.

In this paper we are concerned with the solvability of the nonlinear ordinary differential equation:

$$Dy + f(x,y) = h(x) \tag{1}$$

subjected to the boundary condition:

$$B(y,\dots,y^{(n-1)}) = r. \tag{2}$$

We suppose that

$$D : \operatorname{dom} D \subset C([a,b],\mathbb{R}) \to C([a,b],\mathbb{R})$$

is the $n-$th order linear differential operator defined by

$$(Dy)(x) = \sum_{i=0}^{n} q_i(x)y^{(i)}(x),$$

where $q_i \in C([a,b],\mathbb{R})$, for $i = 0,\dots,n$ and $q_n(x) \neq 0$ for every $x \in [a,b]$, $f : [a,b]\times\mathbb{R} \to \mathbb{R}$ satisfies the Caratheodory conditions; i.e., $f(\cdot,y)$ is measurable for all $y \in \mathbb{R}$, $f(x,\cdot)$ is continuous for almost every $x \in [a,b]$ and, for each $k \geq 0$, there is a function $\psi_k \in L^1([a,b],\mathbb{R}^+)$ such that $|f(x,y)| \leq \psi_k(x)$, for almost every $x \in [a,b]$ and all $|y| \leq k$, $h \in L^1([a,b],\mathbb{R})$.

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