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A NOTE ON A SEMILINEAR ELLIPTIC PROBLEM

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Introduction. Let Ω be a bounded set in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$. In this note, we are concerned with the Dirichlet problem

$$\begin{cases} \Delta^2 u + c\Delta u + b[(u+1)_+ - 1] & \text{on } \Omega, \\ u = 0,\ \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{I}$$

where $u_+ = \max\{u, 0\}$, $b, c \in \mathbb{R}$ and Δ^2 denotes the biharmonic operator.

Equations with nonlinearities of this type have been extensively studied in the contex of *second* order elliptic operators. We refer the reader to a recent paper by Lazer and McKenna [2] for a rather complete account of the results obtained in this direction. Also see the bibliography in [2].

Moreover, in [2] it has been pointed out how these type of nonlinearities also furnish a good model for the study of suspension bridges. Naturally, this leads to the consideration of fourth order elliptic operators ; problem (I), in particular, becomes relevant when studying travelling wave solutions (cf. [3], [2, p. 558]).

We notice that problem (I) always admits the *trivial* solution $u = 0$. Thus, a natural question to ask is whether other solutions exist. In this direction, we have the following theorem.

Theorem 1. *Let $\lambda_1 > 0$ be the first eigenvalue of $-\Delta$ in $H_0^1(\Omega)$ and let $c < \lambda_1$. We have that*

$$\text{problem (I) admits a nontrivial solution} \iff b \geq \lambda_1(\lambda_1 - c).$$

Remark 1. Concerning the existence part of Theorem 1, notice that if $b = \lambda_k(\lambda_k - c)$ and λ_k is an eigenvalue for $-\Delta$ with corresponding eigenfunction Φ_k, then $u = s\Phi_k$ defines a C^∞ solution for (I) for every $s \in \mathbb{R}$ sufficiently small to guarantee that $s\Phi_k(x) > -1$ for all $x \in \Omega$. So, one obtains infinitely many solutions in this case. We shall prove that other solutions exist. More precisely, we have the following theorem.

Theorem 2. *Assume that $b \geq \lambda_1(\lambda_1 - c) > 0$. Then there exists a solution $u = u(x)$ of (I) satisfying*

$$u(x) < 0 \qquad \text{on } \Omega. \tag{0.1}$$

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