Differential and Integral Equations, Volume 5, Number 3, May 1992, pp. 537–552.

STABILITY PROPERTIES FOR SOLUTIONS OF GENERAL EULER–LAGRANGE SYSTEMS*

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(Submitted by: James Serrin)

Abstract. We discuss the global asymptotic stability of the rest state for the Euler - Lagrange systems of ordinary differential equations. The basic assumption is that the systems in question arise from integrals $\mathcal{F}(r,u,u')$ which are convex in the derivative u'. The behavior of the Legendre transform of $\mathcal{F}(r,u,u')$ with respect to u' (namely, in physical terms, the Hamiltonian associated to the Lagrangian $\mathcal{F}$) gives the main tool to obtain our conclusions. The results generalize and extend a number of earlier stability theorems for second order ordinary differential equations and systems.

1. Introduction. Recently Pucci and Serrin in [4-5] have established global asymptotic stability as $r \to \infty$ of the rest state $u \equiv 0$ for weak extremals of both scalar and vector variational problems of the form

$$\delta \int_I g(r)\{G(u') - F(r,u)\}dr = 0, \tag{1.1}$$

where $I = [R,\infty)$ with $R > 0$, $G(0) = F(r,0) = 0$ and the integrand is of class C^1.

The most important conditions in [4-5] are that G is strictly convex on $\mathbb{R}^N$, that $(F_u(r,u),u) > 0$ for r large and $u \neq 0$, and that g is positive and increasing. (Here $(\cdot,\cdot)$ denotes the inner product in $\mathbb{R}^N$.)

The global asymptotic stability of the rest state established in [4-5] extends theorems for second order nonlinear ordinary differential equations due to Levin and Nohel [2] and Artstein and Infante [1] (for the linear case see Smith [6]). The main tool in [4-5] consists in the construction of an appropriate Liapunov function for the Euler–Lagrange system of (1.1), based on the general theory of variational identities proposed by Pucci and Serrin in [3].

In this paper we extend the asymptotic stability results of [4-5] to the general variational problem

$$\delta \int_I g(r)\,\mathcal{F}(r,u,u')dr = 0, \tag{1.2}$$

whose Euler–Lagrange system has the form

$$\{\,g(r)\,\mathcal{F}_p(r,u,u')\}' = g(r)\,\mathcal{F}_u(r,u,u'). \tag{1.3}$$

Received for publication March 1991.

*Research partially supported by the Italian Ministero dell'Università e della Ricerca Scientifica e Tecnologica, fondi 40%.

AMS Subject Classifications: 34D05, 35B35, 93D20.