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NONTRIVIAL SOLUTIONS OF NONLINEAR EQUATIONS FROM SIMPLE EIGENVALUES AND THEIR STABILITY

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Abstract. The purpose of this paper is to study stability and instability of steady-state solutions of the dynamical system $\frac{du}{dt} = F(\lambda, u)$ depending on a parameter, where the nonlinear mapping F is of the form $F(\lambda, u) := T(u) - L(\lambda, u) - H(\lambda, u) - K(\lambda, u)$, T and $L(\lambda, \cdot)$ are continuous linear mappings, and H and K are C^1-mappings. We assume that $(\bar{\lambda}, 0)$ is a solution of $F(\lambda, u) = 0$ and that the mapping $T - L(\bar{\lambda}, 0)$ is a Fredholm operator with index zero and nullity one. We establish theorems on the existence and stability of nontrivial solutions under some additional hypotheses on the mappings. Stability is analyzed via explicit forms of parameter families of nontrivial solutions. The results are illustrated by a class of elliptic partial differential equations.

Introduction. Consider a dynamical system

$$\frac{du}{dt} = g(u),$$

where t is time and u is the state of the system under consideration. The *steady-state* solutions of this system are solutions of the equation $g(u) = 0$. A central problem is to decide whether or not a steady-state solution is stable. To be more specific, suppose that u_0 is a solution of $g(u) = 0$. We perturb (slightly) this solution to the state $u_0 + v$ and consider the initial-value problem:

$$\frac{dw}{dt} = g(w), \quad w(0) = u_0 + v.$$

We want to determine whether there exists a solution $w(t)$ of this problem that tends to u_0 as $t \to \infty$. We consider the linearized problem

$$\frac{dv}{dt} = g'(u_0)v,$$

where $g'(u_0)$ denotes the Fréchet derivative of g at u_0. It is well known that if the spectrum of $g'(u_0)$ lies in the left-half plane, then the solution v decays exponentially to zero as $t \to \infty$. In this case we say that u_0 is an (*asymptotically*) *stable* solution.

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