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A REGULARITY THEORY FOR A MORE GENERAL CLASS OF QUASILINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS AND VARIATIONAL INEQUALITIES

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Abstract. By means of an inequality of Poincaré type, a weak Harnack inequality for the gradient of a solution and an integral inequality of Campanato type, it is shown that solutions to degenerate parabolic variational inequalities are locally Hölder continuous. Using a difference quotient method and Moser type iteration it is then proved that the gradient of a solution is locally bounded. Finally using iteration and scaling it is shown that the gradient of the solution satisfies a Campanato type integral inequality and is locally Hölder continuous.

1. Introduction. In this paper we study regularity of second order degenerate parabolic equations and variational inequalities involving evolutionary p-Laplacian functionals. In [6] Choe established interior regularity for weak solutions of

$$u_t - (a(\nabla u)_i)_{x_i} + b(x,t,u,\nabla u) \geq 0$$

under appropriate assumptions on the competing function class, where a satisfies a standard ellipticity condition and b satisfies a quadratic growth condition on ∇u. Thus it is natural to ask regularity questions for degenerate parabolic cases. Here we prove Hölder continuity of weak solutions and their gradients for degenerate parabolic variational inequalities.

Now we define our problem. Suppose $\Omega \subset \mathbb{R}^N$ is a bounded domain. We assume $\psi \in C^2(Q)$ and $\psi \leq 0$ on the parabolic boundary of Q where $Q = \Omega \times (0,T)$, $T > 0$. We define $K = \{v \in C^0(0,T : L^2(\Omega)) \cap L^p(0,T : W_0^{1,p}(\Omega)); v \geq \psi \text{ a.e. in } Q\}$, where we assume $1 < p < \infty$ naturally.

We say $u \in K$ is a solution to the variational inequality

$$u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) + b(x,t,u,\nabla u) \geq 0 \tag{1.1}$$

if $u \in K$ satisfies the following inequality:

$$\begin{aligned}\int_0^T \int_\Omega \big(v_t(v-u) + |\nabla u|^{p-2}\nabla u \cdot (\nabla v - \nabla u) + b(x,t,u,\nabla u)(v-u)\big)\,dx\,dt \\ + \tfrac{1}{2}\|v(0) - u(0)\|^2_{L^2(\Omega)} \geq 0\end{aligned} \tag{1.2}$$

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