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GLOBAL BREAKING OF SYMMETRY OF POSITIVE SOLUTIONS ON TWO-DIMENSIONAL ANNULI

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Abstract. In this paper, we consider the bifurcation of non-radially symmetric positive solutions from radially symmetric positive solutions of the equation

$$-\Delta u = f(u) \quad \text{in} \quad D,$$
$$u = 0 \quad \text{on} \quad \partial D.$$

Here D is a two dimensional annulus and either f or the inner radius of the annulus is the bifurcation parameter. We obtain many distinct global branches of non-symmetric positive solutions which do not intersect.

We show how a particular cone and the formula for the index of a fixed point in a cone from [3] can be used to obtain much better global information than has been previously obtained on the bifurcating branch of non-symmetric positive solutions of the equation

$$\begin{aligned} -\Delta u &= f(u) \quad \text{in} \quad D, \\ u &= 0 \quad \text{on} \quad \partial D, \end{aligned} \tag{1}$$

where D is an annulus in $\mathbb{R}^2$. Here the bifurcation occurs when we vary f or the thickness of the annulus. (We are taking the outer circle to have radius 1). Under suitable assumptions, we obtain arbitrarily many *global* continuous branches of non-symmetric positive solutions which do not intersect. A number of authors have studied *local* bifurcation of positive non-radially symmetric solutions from positive radially symmetric solutions. See Lin [15] and Smoller and Wasserman [20], where further references can be found. However, at best, their techniques give rather limited information on the global behaviour of bifurcating branches and on the multiplicity of solutions. The cone we introduce is motivated by some work of Benjamin, Bona and Bose [2]. (Indeed, a slight generalization of the index formula in [3] could be used to simplify some of their work). Unfortunately, it is unclear how to generalize our results to more than two dimensions (though we sketch how to obtain one branch of non-symmetric positive solutions in this case).

We intend to give a much more complete discussion of problems on thin and thick annuli elsewhere (especially asymptotic problems). We will show there that

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