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LONG TIME BEHAVIOR OF SOLUTIONS OF THE GENERALIZED BENJAMIN-BONA-MAHONY EQUATION IN TWO SPACE DIMENSIONS

PIOTR BILER
Mathematical Institute, University of Wrocław, pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland
and (1990/91)
Université de Paris-Sud, Laboratoire d'Analyse Numérique, bât. 425, 91405 Orsay, France

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Abstract. It is shown that the supremum norms of the solutions with small initial data of the generalized Benjamin-Bona-Mahony equation $u_t - \Delta u_t = (b, \nabla u) + u^p(a, \nabla u)$ in $x \in \mathbb{R}^2$, $t \in \mathbb{R}^+$, where $0 \neq b$, $a \in \mathbb{R}^2$, $p \geq 3$ is an integer, decay to zero like $t^{-2/3}$ as t tends to infinity. The proof of this result is based on an analysis of the linear equation $u_t - \Delta u_t = (b, \nabla u)$ which is much more difficult than in the one-dimensional case studied by J. Albert in [1] and [2].

Introduction. We consider in this paper the Cauchy problem for the nonlinear dispersive equation

$$u_t - \Delta u_t = (b, \nabla u) + u^p(a, \nabla u), \quad x \in \mathbb{R}^2, \quad t \in \mathbb{R}^+, \quad u(x,0) = u_0(x), \qquad (1)$$

generalizing the Benjamin-Bona-Mahony equation which models long waves of small amplitude in one dimension. Here a and $b \neq 0$ are fixed vectors in $\mathbb{R}^2$, $(\,\cdot\,,\,\cdot\,)$ is the standard inner product in $\mathbb{R}^2 \times \mathbb{R}^2$. The BBM equation

$$u_t - u_{xxt} + (u + u^2/2)_x = 0 \qquad (2)$$

can be regarded as an alternative model for the Korteweg-de Vries equation. The pseudo-parabolic equation (1) is a natural generalization of (2) and it also can be interpreted as a regularization of a single conservation law in several space dimensions.

Results on the existence, uniqueness and regularity of solutions to (1) have been discussed thoroughly by J.A. Goldstein, R. Kajikiya and S. Oharu in a recent paper [4]. However, the authors of [4] have not considered the questions related to asymptotic behavior of solutions to (1) when t tends to infinity. In fact, their equation is more general than (1); $u_t - \Delta u_t = \nabla \cdot F(u)$ with a nonlinear vector function F is considered in open subsets of $\mathbb{R}^n$. It seems that in such a setting a few long time behavior results can be obtained, except for the conservation of the Sobolev space $W^{1,2}(\mathbb{R}^n)$ norm $\|u(t)\|_1$. The reason is that the linear equation $u_t - \Delta u_t = 0$ has

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