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STEADY-STATE SOLUTIONS OF MAXWELL'S EQUATIONS OVER A THREE-DIMENSIONAL CONDUCTING HALF SPACE

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Abstract. A rigorous derivation is given of steady-state solutions $v(x)$ to the problem of the electromagnetic field over a three-dimensional conducting half space $\mathbb{R}^3_-$ due to a time-harmonic source located in $\mathbb{R}^3_+$ ($\mathbb{R}^3_\pm = \{x \in \mathbb{R}^3 : \pm x_3 > 0\}$). For $x \in \mathbb{R}^3_+$, a general solution $v(x)$ has the structure

$$v = v_0 + v_{s_0} + v_{s_+} + v_{s_-} + \overset{\circ}{v}_p + v_p,$$

where v_0 is the static component, v_{s_0} is the AH-wave component, $v_{s_\pm}$ are the surface-wave components, and $\overset{\circ}{v}_p$ and v_p are the spatial-wave components obtained as superpositions of modes having the frequencies of the upper and lower media respectively. For large $|x|$, they decay as follows: v_0, $v_{s_\pm}$, $v_p = O(|x|^{-2})$, $v_{s_0} = o(|x|^{-1})$, $\overset{\circ}{v}_p = O(|x|^{-1})$. An explicit expression is given for the leading term of the asymptotics of $\overset{\circ}{v}_p$. In particular, contrary to popular belief, there is no component which decays like $O(|x|^{-1/2})$ along the interface $\{x_3 = 0\}$.

1. Introduction. For Maxwell's equations over a conducting half space, the solution $u(x,t)$ for quite general sources $f(x,t)$ can be represented by Duhamel's principle [3, p. 197]. Time-harmonic solutions arise as approximations to such solutions for large t when the source is time-harmonic. For such approximations, it is often possible to obtain much more information than for the actual solution. For this reason, it is important to give a rigorous derivation of such solutions and determine their properties.

To formulate the problem let I_3 be the 3×3 identity matrix, let $\underline{O}$ be the 3×3 zero matrix, and let B and E be the diagonal matrices

$$\begin{aligned} B &= \operatorname{diag}(-i\sigma I_3, \underline{O}), \\ E(x) &= \operatorname{diag}[\varepsilon(x) I_3, \mu(x) I_3] = \chi_+(x_3)\operatorname{diag}[\varepsilon_0 I_3, \mu_0 I_3] + \chi_-(x_3)\operatorname{diag}[\varepsilon I_3, \mu I_3] \\ &\equiv E_+ + E_-. \end{aligned}$$

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