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THE NONLINEAR SCHRÖDINGER LIMIT AND THE INITIAL LAYER OF THE ZAKHAROV EQUATIONS

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Abstract. We discuss the formation of the convergence of solutions for the Zakharov equations as the ion sound speed λ goes to infinity. We describe how the solutions tend to the corresponding solutions for the nonlinear Schrödinger equation and how the initial layer phenomena occur. A clear account is given of the relation between the convergence rate in λ and the formation of the initial layers.

1. Introduction and theorems. A simple system of equations describing the propagation of Langmuir turbulence in an unmagnetized, completely ionized hydrogen plasma was first obtained by Zakharov [17] by means of a two-fluid description of the plasma. The system consisting of the ion sound equation and the electric field propagation equation with nonlinear coupling terms is derived from Maxwell's equations and linearized hydrodynamical equations. Another derivation from a Lagrangian formalism is given by Gibbons, Thornhill, Wardrop, and ter Haar [5].

In suitably scaled coordinates, the Zakharov equations are the following:

$$i\frac{\partial E}{\partial t} + \Delta E = nE, \tag{1.1}$$

$$\lambda^{-2}\frac{\partial^2 n}{\partial t^2} - \Delta n = \Delta|E|^2, \tag{1.2}$$

where E and n are functions on the time-space $\mathbb{R} \times \mathbb{R}^N$ with values in $\mathbb{C}^N$ and $\mathbb{R}$, respectively, and $\lambda > 0$ is a parameter. In these equations, E is the slowly varying complex amplitude of the electric field $\mathcal{E}$ of the Langmuir waves with plasma frequency ω_p,

$$\mathcal{E}(t,x) = \mathrm{Re}\,(E(x,t)\exp(-it\omega_p)),$$

n is the deviation of the ion density from its equilibrium, λ is the ion sound speed, the right hand side of (1.1) describes the shift of plasmon frequency caused by the slow density variation n, and the right hand side of (1.2) describes the driving force caused by the pressure of plasmon gas.

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