

BOUNDED PERTURBATIONS WITH MULTIPLE DELAYS OF FORCED HARMONIC OSCILLATORS AT RESONANCE

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Abstract. Sufficient conditions are given for the existence of 2π -periodic solutions for the two functional differential equations $x''(t) + x(t) + h_1(x(t-r)) + h_2(x(t-s)) = p(t)$ and $x''(t) + x(t) + h(x(t-r)) + g(x'(t-s)) = p(t)$, where h, h_1, h_2 , and g are all bounded, continuous real-valued functions having asymptotic limits at $\pm\infty$, p is a 2π -periodic real-valued function, and r, s are fixed real constants, $0 \leq r, s < 2\pi$. These results are extensions to functional differential equations with multiple delays of the results obtained by Lazer and Leach for bounded perturbations of forced harmonic oscillators at resonance and those by Nagle and Sinkala for similar problems where the perturbations involve the derivative.

1. Introduction. We consider the existence of 2π -periodic solutions for the two functional differential equations

$$x''(t) + x(t) + h_1(x(t-r)) + h_2(x(t-s)) = p(t) \quad (1.1)$$

and

$$x''(t) + x(t) + h(x(t-r)) + g(x'(t-s)) = p(t), \quad (1.2)$$

where h, h_1, h_2 , and g are all bounded, continuous real-valued functions having asymptotic limits at $\pm\infty$, p is a 2π -periodic real-valued function, and r, s are fixed real constants, $0 \leq r, s < 2\pi$.

Here we attempt to extend to functional differential equations with multiple delays the results obtained by Lazer and Leach [14] for bounded perturbations of forced harmonic oscillators at resonance and those by Nagle and Sinkala [19] for similar problems where the perturbations involve the derivative. Our approach uses an abstract result of Nagle and Sinkala [19] which is in the spirit of the approach used by Cesari [3], Cesari and Kannan [4], and Mawhin [17].

There are few results known for nonlinear boundary value problems at resonance involving multiple delays, or even for a single delay where the dimension of the kernel of the linear operator is greater than one. For results involving periodic solutions of second order functional differential equations with a single delay, and where the kernel consists typically of the constant functions, we refer the reader to the papers of De Pascale and Iannacci [7], Iannacci and Nkashama [12, 13], Mawhin [15, 16], Metzen [18], Nussbaum [21], and Perello [22] (where the delay term contains a small parameter), the monographs of Bellman and Cooke [1], Burton [2], and Hale

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