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A SEMIGROUP APPROACH TO THE TIME DEPENDENT PARABOLIC INITIAL-BOUNDARY VALUE PROBLEM

Alessandra Lunardi
Dipartimento di Matematica, Università di Cagliari, Via Ospedale 72, 09124 Cagliari, Italy

Eugenio Sinestrari
Dipartimento di Matematica, Università di Roma "La Sapienza"
P. Aldo Moro 2, 00185 Roma, Italy

Wolf von Wahl
Lehrstuhl für Angewandte Mathematik, Universität Bayreuth
P.O. Box 101251, 8580 Bayreuth, Germany

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Abstract. In this paper, a general initial-boundary value problem for a higher order linear parabolic equation with time dependent coefficients and nonhomogeneous data is studied via the abstract results obtained from a generalization of the classical theory of analytic semigroups of linear operators.

1. Introduction. This paper gives a proof of the existence, uniqueness, and optimal Hölder regularity of the solutions of the linear parabolic initial-boundary value problem of higher order

$$\begin{cases} D_t u(t,x) = \sum_{|\gamma|\le 2m} a_\gamma(t,x) D_x^\gamma u(t,x) + f(t,x), \\ \qquad\qquad (t,x)\in Q = [0,T]\times\bar{\Omega}, \\ u(0,x) = u_0(x),\ x\in\bar{\Omega},\ \sum_{|\beta|\le m_j} b_{j\beta}(t,x) D_x^\beta u(t,x) = g_j(t,x), \\ \qquad\qquad (t,x)\in S = [0,T]\times\Gamma, \quad j=1,\dots,m. \end{cases} \tag{1}$$

Here Ω is a bounded open set in $\mathbb{R}^n$ with smooth boundary Γ. For every multi-index $\beta = (\beta_1, \dots \beta_n) \in \mathbb{N}^n$, we set $|\beta| = \beta_1 + \dots + \beta_n$ and $D_x^\beta u = D_{x_1}^{\beta_1} \dots D_{x_n}^{\beta_n} u$. We assume that for every $t\in[0,T]$, the operator

$$A(t) = \sum_{|\gamma|\le 2m} a_\gamma(t,\cdot) D_x^\gamma \tag{2}$$

is elliptic, and the boundary operators

$$B_j(t) = \sum_{|\beta|\le m_j} b_{j\beta}(t,\cdot) D_x^\beta, \qquad j=1,\dots,m, \tag{3}$$

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