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THE BLOW-UP BEHAVIOR OF THE SOLUTION OF AN INTEGRODIFFERENTIAL EQUATION

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Abstract. In this paper, the method of subsolution and the comparison principle are applied to the study of the blow-up behavior of the solution of an integrodifferential equation arising from the nuclear reactor dynamics. We give a blow-up criterion and study the blow-up set and the blow-up rate.

1. Introduction.

In this paper, we discuss the blow-up property of the problem

$$u_t - \Delta u = \mu(x)\Big\{p\exp\Big[\int_0^t \int_\Omega \beta(y)u(y,\tau)\,dy\,d\tau\Big] - 1\Big\}, x \in \Omega, t > 0, \tag{1.1}$$

$$u(x,0) = \phi(x), \quad x \in \Omega, \tag{1.2}$$

$$u(x,t) = 0, \quad x \in \partial\Omega, \quad t > 0. \tag{1.3}$$

We always assume that

(1) Ω is an open bounded smooth domain in $\mathbb{R}^n$,
(2) ϕ is a smooth function such that $\phi(x) = 0$, $x \in \partial\Omega$, and $\phi(x) \geq 0$,
(3) μ is a uniformly Hölder continuous function such that $\mu(x) \geq 0$ and $\mu(x) \not\equiv 0$,
(4) β is a nonnegative continuous function such that $\beta(x) \not\equiv 0$,
(5) $p > 1$.

Problem (1.1–3) occurs in nuclear reactor dynamics in which u represents the incremental temperature from zero and the term

$$g(t) \equiv \int_\Omega \beta(x)u(x,t)\,dx \tag{1.4}$$

measures the increment temperature feedback reactivity. Let

$$h(t) = p\exp\Big[\int_0^t g(\tau)\,d\tau\Big] - 1. \tag{1.5}$$

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