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A MODEL OF WAVE PROPAGATION IN A NONLINEAR SUPERCONDUCTING DIELECTRIC

HAMID BELLOUT and FREDERICK BLOOM†
Department of Mathematical Sciences, Northern Illinois University, DeKalb, IL 60115

JINDRICH NECAS‡
Faculty of Mathematics, Charles University, Prague, Czechoslovakia

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Abstract. The problem of a linearly polarized plane electromagnetic wave propagating in an unbounded domain occupied by a nonlinear dielectric is considered under the assumption that the conduction current is inversely proportional to the magnetic permeability, and directly proportional to the electric field strength. We prove that, in the limit of vanishing magnetic permeability, the electromagnetic wave tends to one which propagates with infinite speed; the demonstration hinges upon proving that the globally existing solutions of a nonlinearly damped, nonlinear wave equation converge weakly in L^2 to the unique classical solution of an associated heat equation.

1. Introduction. We consider in this paper the problem of a linearly polarized plane electromagnetic wave of the form

$$\begin{aligned} \underset{\sim}{E} &= (0, E(x,t), 0), \quad \underset{\sim}{D} = (0, D(x,t), 0) \\ \underset{\sim}{B} &= (0,0,B(x,t)), \quad \underset{\sim}{H} = (0,0,H(x,t)) \end{aligned} \tag{1.1}$$

propagating in an unbounded cylindrical domain $\Omega \subset \mathbb{R}^3$ with generators parallel to the x-axis of a fixed Cartesian coordinate system chosen in Ω. The fields $\underset{\sim}{E}, \underset{\sim}{D}, \underset{\sim}{B}, \underset{\sim}{H}$ displayed in (1.1) are, respectively, the electric, electric displacement, magnetic, and magnetic intensity fields; throughout Ω, these fields must satisfy Maxwell's equations which, with the assumption of zero free charge density, assume the form

$$\begin{aligned} \frac{\partial \underset{\sim}{B}}{\partial t} &= - \text{ curl } \underset{\sim}{E}, \qquad \text{div } \underset{\sim}{B} = 0, \\ \frac{\partial \underset{\sim}{D}}{\partial t} + \underset{\sim}{J} &= \text{ curl } \underset{\sim}{H}, \quad \text{div } \underset{\sim}{D} = 0, \end{aligned} \tag{1.2}$$

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‡On Leave in the Department of Mathematical Sciences, Northern Illinois University.
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