Differential and Integral Equations, Volume 5, Number 5, September 1992, pp. 1151–1184.

REGULARITY PROPERTIES OF THE EVOLUTION OPERATOR FOR ABSTRACT LINEAR PARABOLIC EQUATIONS

PAOLO ACQUISTAPACE
Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate
Università di Roma "La Sapienza", 00161 Roma, Italy

BRUNELLO TERRENI
Dipartimento di Matematica "F. Enriques", Università di Milano, 20133 Milano, Italy

(Submitted by: G. Da Prato)

0. Introduction. Consider the parabolic Cauchy problem

$$u'(t) - A(t)u(t) = f(t), \quad t \in [s,T], \quad u(s) = x \tag{0.1}$$

in a general Banach space E, where the operators $A(t)$ generate analytic semigroups in E, with possibly non-dense domains. The existence of the evolution operator $U(t,s)$ for problem (0.1) is guaranteed provided the family $\{A(t)\}_{t\in[0,T]}$ enjoys some regularity with respect to t. It is shown in [5, §7] that all kinds of hypotheses used in the literature can be essentially reduced to two independent sets of assumptions which are weaker than any other: namely, the classical ones by Kato and Tanabe [10], revisited in [3, 4], and those introduced in [5] and used in [6, 1].

The properties of $U(t,s)$ and its regularity with respect to t are very well known now. Much less information is available on the regularity of $s \to U(t,s)$; the only classical result is that

$$\exists \left[\tfrac{d}{d\sigma}U(t,\sigma)x\right]_{\sigma=s} = -U(t,s)A(s)x, \quad \forall x \in D_{A(s)}, \quad \forall 0 \le s \le t \le T, \tag{0.2}$$

provided all domains $D_{A(t)}$ are dense in E (see e.g., [17, Theorems 5.2.1 and 5.3.3]); some improvements of (0.2) can be found in [9, Theorem I], [16, §1.11]. In addition, under the assumptions of [10], it is known that the operator-valued function $s \to U(t,s)$ is differentiable in $\mathcal{L}(E)$ for $s < t$, and $dU(t,s)/ds$ is a bounded extension of the closed operator $-U(t,s)A(s)$ (see [17, Theorem 5.3.3]). In a recent paper by Lunardi [13], the latter property, with several related results, has been shown to be true if the $A(t)$'s have a (possibly non-dense) common domain D and satisfy strong regularity assumptions.

The goal of this paper is a systematic study of the properties of $s \to U(t,s)$ under the assumptions of [3]; in this case, our results extend those of [13] and seem to be optimal. We also study the same problem under the assumptions of [5], but the situation here is more complicated and requires additional assumptions.

Received for publication December 3, 1990.
AMS Subject Classifications: 34G10, 47D05, 58D25.