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# HOMOCLINIC ORBITS FOR A CLASS OF HAMILTONIAN SYSTEMS

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**Abstract.** Using a new compact imbedding theorem, we prove the existence of infinitely many solutions in $H^1(\mathbb{R})$ of the system

$$\ddot{q} - L(t)q + W_q(t,q) = 0$$

when $W_q(t,\cdot)$ is even, superquadratic at infinity and subquadratic at the origin.

**1. Introduction.** This paper deals with the existence of nontrivial solutions $q \in H^1(\mathbb{R},\mathbb{R}^n)$ of the system

$$\ddot{q} + V_q(t,q) = 0. \tag{HS}$$

Those solutions are called homoclinic orbits emanating from 0.

P.H. Rabinowitz and K. Tanaka [4] have recently shown that the Hamiltonian system (HS) possesses a homoclinic orbit emanating from 0 by using a variant of the "Mountain Pass" theorem relying on Ekeland's Variational Principle. The use of this method is due to the difficulty of verifying the Palais-Smale condition. In this paper, we show that the Palais-Smale condition is satisfied and we use the usual Mountain Pass Theorem to prove the result of Rabinovitz and Tanaka. Moreover, if $W(t,\cdot)$ is an even function, we prove the existence of an unbounded sequence of homoclinic orbits of (HS) emanating from 0 by using the "symmetric" mountain pass theorem.

Throughout the paper, it will be assumed that $V$ satisfies

(V1) $V(t,x) = -\frac{1}{2}L(t)x\cdot x + W(t,x)$,

(V2) $L(t) \in C(\mathbb{R},\mathbb{R}^{n^2})$ is a positive definite symmetric matrix for all $t \in \mathbb{R}$, and there is continuous function $\alpha\colon \mathbb{R} \to \mathbb{R}$ such that $\alpha(t) > 0$ for all $t \in \mathbb{R}$ and $L(t)x\cdot x \geq \alpha(t)|x|^2$,

(V3) $W \in C^1(\mathbb{R}\times\mathbb{R}^n,\mathbb{R})$ and there is a constant $\mu > 2$ such that

$$0 < \mu W(t,x) \leq x\cdot W_q(t,x)$$

for all $x \in \mathbb{R}^n \setminus \{0\}$ and $t \in \mathbb{R}$,

(V4) $W_q(t,x) = o(|x|)$ as $x \to 0$ uniformly in $t \in \mathbb{R}$.

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