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ENERGY DECAY FOR DIRICHLET PROBLEMS IN IRREGULAR DOMAINS WITH QUADRATIC HAMILTONIAN

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Abstract. Let u be a solution of the formal Dirichlet problem

$$\begin{cases} Lu \equiv -\partial_j(a_{ij}\partial_i u) = f(x,u,Du) & \text{in } \Omega - E \\ u = h & \text{in } E \end{cases}$$

with $h \in H^1$, E a bounded Borel subset of $\mathbb{R}^n$, f a Caratheodory function growing quadratically in the gradient variable and L a uniformly elliptic operator. We estimate the energy decay and give sufficient conditions on h and E to guarantee the continuity of u in E through the investigation of two-obstacle problems. Similar results are also proved for the one-obstacle case.

1. Introduction. Let E be a bounded Borel set of $\mathbb{R}^n$, $n \geq 3$, and Ω a regular open bounded domain such that $\overline{E} \subset \Omega$. The formal "irregular Dirichlet problem,"

$$\begin{cases} \Delta u = 0 & \text{on } \Omega - E \\ u = h & \text{on } E, \end{cases} \tag{1.1}$$

where $h \in H^1(\Omega)$, can be given the local variational meaning

$$\begin{cases} u \in H^1(\Omega), & u = h \quad \text{q.e. on } E \\ \int_\Omega Du\, Dw\, dx = 0 & \\ \forall w \in H_0^1(\Omega), & w = 0 \quad \text{q.e. on } E, \end{cases} \tag{1.2}$$

where "q.e. on E" means everywhere but on a set of capacity 0 (we will recall the notion of capacity in 2.1); the pointwise values of u and w are to be considered for the quasi continuous representative of these H^1 functions (see (2.5)).

Also, following G. Dal Maso, U. Mosco and M.A. Vivaldi [10], for $h \in H^1(\Omega)$, we can consider u as the variational solution of the two-obstacle problem

$$\begin{cases} \psi_1 \leq u \leq \psi_2 & \text{q.e. in } \Omega, \quad u \in H^1(\Omega) \\ \int_\Omega Du\, D(u-v)\, dx \leq 0 & \\ \forall v\colon \psi_1 \leq v \leq \psi_2 & \text{q.e. in } \Omega, \quad u - v \in H_0^1(\Omega), \end{cases} \tag{1.3}$$

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