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THE CONCEPT OF VALUE IN DIFFERENTIAL GAMES OF SURVIVAL AND VISCOSITY SOLUTIONS OF HAMILTON-JACOBI EQUATIONS

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Abstract. We study the survival game problem without any restriction on its duration. We prove, using a weak limit procedure, that the values in the different formulations of Elliott-Kalton, Fleming and Friedman exist and are viscosity solutions of the Hamilton-Jacobi-Isaacs equation with suitable boundary conditions. We use the uniqueness theorems in [6-7] for such free boundary Dirichlet problems to prove the equivalence of the three definitions.

0. Introduction. In this paper we study a dynamical system controlled by two players

$$y' = f(y, a, b), \quad y(0) = x \in \mathbb{R}^N. \tag{0.1}$$

We assume that the vector field $f\colon \mathbb{R}^N \times A \times B \to \mathbb{R}^N$ is continuous and satisfies a uniform Lipschitz condition in the state variable

$$|f(x,a,b) - f(z,a,b)| \leq L|x-z| \quad \text{for all} \quad x, z, a, b.$$

We also suppose that the control sets A, B are compact subsets of $\mathbb{R}^M$ and consider the following sets of admissible controls:

$$\mathcal{A} := \{a\colon \mathbb{R}_+ \to A \,\text{measurable}\},\ \mathcal{B} := \{b\colon \mathbb{R}_+ \to B \,\text{measurable}\}.$$

We will denote by $y_x(\cdot)$ or simply by $y(\cdot)$ a solution of the system (0.1) corresponding to a choice of a and b. We are also given a closed "target" (or "terminal") set $\mathcal{T} \subset \mathbb{R}^N$ and the two following functions: a running cost $h\colon \mathbb{R}^N \times A \times B \to \mathbb{R}$ which we will suppose to be continuous and strictly positive, i.e.,

$$h(x,a,b) \geq h_o > 0 \quad \text{for all} \quad x, a, b, \tag{0.2}$$

and a final cost $g\colon \mathcal{T} \to [G, +\infty[$ which will be bounded below ($G > -\infty$) and continuous. For simplicity of notations we assume that in (0.2) $h_o = 1$.

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