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COMPACTNESS AND SPECTRAL PROPERTIES FOR POSITIVE TRANSLATION SEMIGROUPS ASSOCIATED TO MODELS OF POPULATIONS DYNAMICS

L. ALAOUI AND O. ARINO
Department de Mathematiques, Universite de Pau, France

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0. Introduction. Translation semigroups cover a large class of problems including functional differential equations. A systematic study of such semigroups was undertaken a few years ago by A.T. Plant [12], F. Martello [10]. Their importance has been recently recognized in relation to models of populations dynamics. In the context of spaces of the type $E = L^1((-r,0),F)$, they are characterized by A. Grabosch [8] as those semigroups whose generator A is just $Af = \frac{d}{dt}f$ for $f = f(\cdot) \in E$ and the domain $D(A)$ is the part of $W^{1,1}((-r,0),F)$ which satisfies a linear equation $f(0) = \phi(f)$, where ϕ is a bounded linear operator from E into F. Thus, a particular translation semigroup can be characterized amongst others by the operator ϕ with defines its domain.

In [8], Grabosch proved that a number of qualitative properties can be attached directly to the translation semigroup structure and are independent of the particular features possessed by such an example. A fundamental assumption for our main result (Theorem 1 below) is the eventual compactness of the semigroup. Grabosch stated the following general condition to yield this property: ϕ is compact and the operator $\phi \circ A$ has a continuous extension to the whole space. Despite its generality, this condition has limitations. For example, it does not apply to the semigroup considered by O. Arino and M. Kimmel in [2].

Our purpose in this paper is to present further generalizations of Grabosch's condition.

We start by giving some results on the existence and positivity of the semigroup $(T_\phi(t))_{t\geq 0}$ associated with the operator A_ϕ defined on the space $E := L^p((-r,0),F)$, $r > 0, 1 \leq p < \infty$, F being a Banach space, by

$$A_\phi f = f', \quad f \in D(A_\phi) := \{f \in W^{1,p}([-r,0],f) : f(0) = \phi(f)\},$$

where ϕ is a bounded linear operator from E into F. Those results are the same as those obtained by Grabosch in the case $p = 1$ with some minor modifications in their proofs.

Next, we will address the problem of compactness for the semigroup $(T_\phi(t))_{t\geq 0}$ and indicate several hypotheses allowing us to pass from "ϕ compact" to "$(T_\phi(t))_{t\geq 0}$ eventually compact."

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