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A QUASILINEAR INTEGRODIFFERENTIAL EQUATION OF HYPERBOLIC TYPE

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Abstract. Using energy estimates we discuss global existence and uniqueness of solutions to the integrodifferential equation

$$u_{tt}(x,t) = m(\|\nabla u\|^2)\Delta u(x,t) + \int_0^t a(t-\tau)n(\|\nabla u\|^2)\Delta u(x,\tau)\,d\tau + b(t)u_t + c(t)u + d(x,t).$$

It is shown that if the initial data is $(-\Delta)$-analytic, then the equation above possesses a unique globally defined classical solution. Our results extend previous results in [19] to the degenerate case, and the results in [1] to materials with memory.

1. Introduction. This article is concerned with the study of a second order degenerate integro-differential equation

$$\begin{aligned} u_{tt} &= m\big(\|\nabla u\|^2_{L^2(\Omega)}\big)\Delta u + \int_0^t a(t-\tau)n\big(\|\nabla u\|^2_{L^2(\Omega)}\big)\Delta u d\tau \\ &\quad + b(t)u_t + c(t)u + d(x,t), \\ u &= u^0, \quad u_t = u^1 \text{ for } t=0,\ x\in\Omega, \\ u &= 0 \text{ on } \partial\Omega\times[0,T], \end{aligned} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. The functions a, b, c, d, m, and n are real-valued and satisfy certain conditions to be specified below; $T>0$ is an arbitrary fixed (finite) real number, and, as usual,

$$u_{tt} = \frac{\partial^2 u}{\partial t^2}, \quad \|\nabla u\|^2_{L^2(\Omega)} = \sum_{i=1}^N \int_\Omega \left|\frac{\partial u}{\partial x_i}\right|^2 dx \text{ and } \Delta u = \sum_{i=1}^N \frac{\partial^2 u}{\partial x_i^2}.$$

In the special case where $N=1$, $m(r)=\alpha+r$ (α a positive constant), and $a\equiv 0$, the nonlinear wave equation in (1.1) has occurred while studying the dynamic equilibrium of nonlinear elastic strings. Here, the tension has been assumed to be uniform along the string but may vary with time to accommodate changes in the arc length of the string (see, *e.g.*, Carrier [3], Dickey [8], Narasimha [12], Nishida [13], and Oplinger [16]).

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