Differential and Integral Equations, Volume 6, Number 2, March 1993, pp. 383–391.

ON DIRICHLET PROBLEM IN MORREY SPACES

G. Di Fazio
Dipartimento di Matematica, Universita' di Catania, Viale A. Doria 6 95100 Catania, Italy

(Submitted by: Luis A. Caffarelli)

0. Introduction. In recent years many papers (see e.g. [1, 3, 11, 7]) have been devoted to the study of the continuity properties of solutions to equations of the form

$$-\Delta u + Vu = f, \tag{$*$}$$

or, more generally,

$$Lu + Vu = f, \tag{$**$}$$

where L is a uniformly elliptic operator in divergence form defined by

$$L \equiv -\frac{\partial}{\partial x_j}\Big(a_{ij}\frac{\partial}{\partial x_i}\Big).$$

The novelty, starting with the pioneering work of Aizenman and Simon (see [1]), is the assumption made on the potential term V and the known term f which belongs to the *Stummel-Kato class*, and not the requirement of high integrability, V, $f \in L^p(\Omega)$, $p > n/2$, as in previous works (see e.g. [10], [12]).

We recall that the *Stummel-Kato class* $S(\Omega)$ is the set of the locally integrable functions f such that

$$\lim_{\varepsilon\to 0}\sup_{x\in\Omega}\int_{\{y\in\Omega:\ |y-x|<\varepsilon\}} f(y)|y-x|^{2-n}dy = 0.$$

In the paper [4] we studied the relation of the *Stummel-Kato class* with the scale of *Morrey spaces* $L^{1,\lambda}(\Omega)$ $(0<\lambda<n)$ (for definitions see Section 1). More precisely if $S(\Omega)$ denotes the *Stummel-Kato class*, we have

$$L^{1,\lambda}(\Omega) \subseteq S(\Omega) \subseteq L^{1,\mu}(\Omega) \qquad 0<\mu\le n-2<\lambda<n.$$

In the same paper, assuming V in $L^{1,\lambda}(\Omega)$ $(n-2<\lambda<n)$ and $f\equiv 0$ we proved the local Hölder continuity of the solutions of equation ($**$).

The present paper is concerned with the Dirichlet problem for equation

$$Lu - (b_j u)_{x_j} = (f_j)_{x_j} \tag{$***$}$$

Received for publication October 1991.
AMS Subject Classification: 35B45, 35B65, 35J10.