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ON SOME VECTOR-VALUED NONLINEAR VARIATIONAL PROBLEMS: VARIATIONS ON THE SKYRME-HARTREE-FOCK MODEL IN NUCLEAR PHYSICS

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Abstract. In this paper, we study some variational problems in nuclear physics consisting of minimizing an energy functional depending on several wave functions. We suppose that the nucleons interact through a simplified version of Skyrme's force. And we are interested in the existence of a minimum (a ground state) insuring the stability of the nucleus.

1. Introduction. The purpose of this paper is to study a class of vector-valued minimization problems arising in simplified models of nuclear physics. This work is motivated by a previous one by D. Gogny and P.L. Lions [3]. We refer the reader to their paper and to D. Vautherin [10] for a more detailed description of the physical background and, in particular, for a justification of the model we introduce below.

From a physical viewpoint, we are interested in the stability of a nucleus consisting of N nucleons (except in the last section, we will make no distinction in the sequel between neutrons and protons). Therefore, we want to prove the existence of a ground state for this system. The model we consider is set in the framework of the mean-field approximation and the nucleons are supposed to interact through a schematic version of Skyrme's potential containing only a zero-range two-body attraction and a zero-range three-body repulsion term of the form $\alpha\rho^2 - \beta\rho$, where ρ is the density of the nucleons. Other powers of the density are relevant in nuclear physics like, for example, $\alpha\rho^{2+\frac{2}{3}}$. This is the reason why we will, in the sequel, consider a large class of powers. One should then fit α and β to match the empirical values of the binding energy per nucleon and the saturation density of the nuclear matter (see [10]). But, more generally, we will allow any $\alpha \geq 0$ and $\beta > 0$.

Finally, we will deal with the functional

$$\mathcal{E}(u_1; u_2; \dots; u_N) = \frac{1}{2}\int_{\mathbb{R}^3} \tau\, dx + \frac{\alpha}{2p}\int_{\mathbb{R}^3} \rho^p\, dx - \frac{\beta}{2q}\int_{\mathbb{R}^3} \rho^q\, dx, \tag{1}$$

where $\tau = \sum_{i=1}^N |\nabla u_i|^2$, $\rho = \sum_{i=1}^N |u_i|^2$, $N \geq 1$, $\alpha \geq 0$, $\beta > 0$ and $1 < q < p \leq 3$; the $(u_i)_{1\leq i\leq N}$ are functions in $H^1(\mathbb{R}^3;\mathbb{R})$ or $H^1(\mathbb{R}^3;\mathbb{C})$ which represent the wave functions of the N nucleons.

In fact, comparing with Skyrme's original model (where $q = 2$, $p = 3$, $\alpha > 0$), we have neglected terms like $\int_{\mathbb{R}^3} |\nabla\rho|^2\, dx$, $\int_{\mathbb{R}^3} \rho\tau\, dx$ (see [3]). But, we believe that our

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