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EXISTENCE OF POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS IN $\mathbb{R}^N$

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(Submitted by: P.L. Lions)

Abstract. We study the existence of positive solutions of semilinear elliptic equations in $\mathbb{R}^N$. By using the concentration-compactness argument and the minimax principle, we obtain a positive solution given by some "higher" critical point.

1. Introduction. We consider the existence of positive solutions of the equation

$$\begin{cases} -\Delta u + u = f(x,u) & \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{1}$$

where $N \geq 3$, $f(x,t) \in C(\mathbb{R}^N \times [0,+\infty))$, $f(x,t)$ is continuously differentiable with respect to t.

Suppose $f(x,t) \to \bar{f}(t)$ as $|x| \to \infty$; the equation "at infinity" associated with (1) is

$$\begin{cases} -\Delta u + u = \bar{f}(u) & \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N). \end{cases} \tag{2}$$

P.L. Lions [11] has studied the following minimization problem closely related to (1):

$$I = \inf\{I(u) \mid u \in \mathcal{M}\}, \tag{3}$$

where

$$I(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + u^2 - \int_{\mathbb{R}^N} F(x,u), \tag{4}$$

$$F(x,t) = \int_0^t f(x,s)\,ds, \tag{5}$$

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