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FREDHOLM-TYPE RESULTS FOR INTEGRODIFFERENTIAL IDENTIFICATION PARABOLIC PROBLEMS

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Abstract. We consider linear and nonlinear identification problems for integrodifferential abstract equations. We determine sufficient conditions for uniqueness to imply existence and stability in the linear case and stability in the nonlinear one.

0. Introduction. The problem of determining unknown parameters in differential equations (conventionally named as an *inverse problem*) has drawn the attention of many researchers. When looking for an unknown parameter, together with the usual data, we are given also a (so-called) *additional information*, which can be expressed by an operator of the solution $u : \Phi(u) = g$ (cf. [10], [11]). For instance, when dealing with parabolic equations, operator Φ is sometimes of the following form: $\Phi(u) = u(T, \cdot)$ (*final determination*, cf. [1], [11]). In that case we look for a pair of functions $u(t, x)$ and $z(x)$ (denoted in the abstract case simply by $u(t)$ and z) and we deal with the following two cases:

i) *the linear case*: the parameter z is an unknown coefficient in the free member $E(t)z$, where $E(t)$ is a prescribed operator (in the concrete case we consider the multiplication operator $E(t, x)z(x)$).

ii) *the nonlinear case*: the parameter z is an unknown coefficient in the equation.

We observe that the form of the free term $E(t, x)z(x)$, $E(t, x)$ being a prescribed function, was first introduced in [10] for the elliptic case and in [11] for evolution equations. Such a form of the free member allows one to investigate the uniqueness and the stability of the inverse problem and to reduce it to a *linear one*. As is well known, inverse problems are *ill-posed*. Consequently, in order to choose suitable functional classes, it is necessary to introduce additional assumptions, which make such problems *well-posed*. In our case such assumptions make our problem a

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