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ON THE STABILITY OF PARALLEL SHEAR FLOW OF AN OLDROYD B FLUID

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Abstract. It is proved that plane Couette flow of an Oldroyd B fluid is linearly stable as long as all eigenvalues remain in the left half plane at a strictly positive distance from the imaginary axis.

1. Introduction. It is well known that a system of linear ODEs, $\dot{\mathbf{y}} = \mathbf{A}\mathbf{y}$, is stable if all eigenvalues of $\mathbf{A}$ are in the left half-plane. An analogous result holds for abstract evolution problems, under the assumption that the operator $\mathbf{A}$ generates an analytic semigroup. On the other hand, if $\mathbf{A}$ generates only a C_0-semigroup, then no such result is known; indeed there are counterexamples [3].

The Stokes operator generates an analytic semigroup, and hence the linear stability of flows of Newtonian fluids can be determined by computing the spectrum of the linearization. Non-Newtonian fluids with memory, however, do generally not lead to evolution problems associated with analytic semigroups. No abstract result exists which would assert that the customary procedure of calculating eigenvalues is indeed valid in assessing the stability of flows of such fluids.

In an earlier paper [6], the linear stability of Couette flow of an upper convected Maxwell fluid was proved under the assumption of creeping flow. This extended a result of Gorodtsov and Leonov [1], who showed that the eigenvalues are in the left half plane. The result however, does not allow any claim of stability for non-zero Reynolds number, however small. Indeed, instability for any non-zero Reynolds number was claimed in [1], but this claim was shown to be erroneous [5].

In the present paper, we consider plane Couette flow of an Oldroyd B fluid. This model is like the upper convected Maxwell fluid, but with an additional Newtonian contribution to the viscosity. For this fluid, stability of parallel shear flow is known for sufficiently small Reynolds and Weissenberg numbers, see [2]. Our result is of a different nature, namely it establishes linear stability under the (unverified) assumption that the eigenvalues lie in the left half plane. For Poiseuille flow, this is certainly not the case for high enough Reynolds numbers, and the critical Reynolds number for the Oldroyd B fluid is lower than that for the Newtonian case [4]. For Couette flow, the flow is always stable in the Newtonian case [7], and numerical calculations suggest that this may be so for the Maxwell and Oldroyd B models as well [5].

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