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ALMOST PERIODIC SOLUTIONS OF VOLTERRA EQUATIONS

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Dedicated to the memory of Peter Hess

Abstract. The asymptotic behavior of solutions of the linear abstract Volterra integral equation $u'(t) = Au(t) + (dB * u)(t) + f(t)$ (*) is investigated, where A is a closed linear operator in a Banach space, $B(t)$ is a suitable operator-valued function and f is an almost periodic X-valued function. It is shown that if the real spectrum of equation (*) is countable, then every bounded uniformly continuous solution is almost periodic, provided one of the following conditions is fulfilled: (i) $X \not\supseteq c_0$; (ii) the range of u is weakly relatively compact; (iii) the function $e^{-\lambda t}u(t)$ has uniformly convergent means for each $\lambda \in i\mathbb{R}$. Analogous results are also obtained for differential equations of higher order. Some examples are given to illustrate possible applications of the results to the heat equations in materials with memory and in population dynamics.

1. Introduction. Consider a Volterra integral equation of the type

$$u'(t) = Au(t) + \int_0^\infty dB(\tau)u(t-\tau) + f(t), \tag{1}$$

where A is a closed linear operator in a Banach space X, $\{B(t)\}_{t\geq 0}$ is a family of closed linear operators in X which satisfies some suitable conditions (see below), and $f : \mathbb{R} \to X$ is a continuous function. The theory of these equations up to some point is known. Namely, it is known that, given the history function $u_-(t)$ for $t \leq 0$, solutions of equation (1) such that $u(t) = u_-(t)$ for $t \leq 0$ can be reduced to solutions of the equation

$$u'(t) = Au(t) + \int_0^t dB(\tau)u(t-\tau) + g(t), \quad t \geq 0, \tag{2}$$

where

$$g(t) = f(t) + \int_t^\infty dB(\tau)u_-(t-\tau),$$

the existence of which has been investigated by many authors (see, e.g., [6, 7, 8], [18]).

In this paper, we are concerned with conditions which guarantee almost periodicity of a bounded solution of equation (1). Note that almost periodicity of solutions of

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