Differential and Integral Equations, Volume 7, Number 4, July 1994, pp. 921–948.

STABILITY OF HIGHER NORMS IN TERMS OF ENERGY-STABILITY FOR THE BOUSSINESQ-EQUATIONS: REMARKS ON THE ASYMPTOTIC BEHAVIOUR OF CONVECTION-ROLL-TYPE SOLUTIONS

YOSHIYUKI KAGEI†

Department of Applied Science, Faculty of Engineering
Kyushu University 36, Fukuoka 812, Japan

WOLF VON WAHL

Department of Mathematics, University of Bayreuth, 95440 Bayreuth, Germany

Dedicated to the memory of Peter Hess

Abstract. In an infinite layer heated from below we perturb a steady viscous incompressible fluid flow. The steady flow is assumed to be stable in the sense of Lyapunov with respect to the norm $\sup_{t\geq 0} E^{1/2}(t)$, where $E(t)$ is the kinetic energy of the perturbation at time t. Energy-stability may also set in at later times. Then the steady flow turns out to be stable in the sense of Lyapunov also with respect to L^2-norms of higher order derivatives. In terms of these we are able to indicate absorbing sets for the disturbances if their initial values are small. The smallness required is discussed. As a by-product we obtain that (nonlinear) instability of any steady flow with respect to higher order norms of the disturbances, as considered in [1] by Galdi and Padula for perturbations of the motionless state in various situations, implies (nonlinear) instability with respect to kinetic energy, at least. Finally we consider some aspects of the time evolution of two-dimensional solutions of the Boussinesq-equations. Two-dimensional means that the velocity-field and the temperature do not depend on one of the plane space-variables. We speak of convection-roll-type solutions.

0. Introduction, notations. The operators occurring in the Boussinesq-equations. We consider an infinite layer $\mathbb{R}^2 \times (-\frac{1}{2}, \frac{1}{2})$ heated from below. The layer is filled with an incompressible viscous fluid. The solenoidal vector fields $\underline{u}$ being periodic in $(x, y) \in \mathbb{R}^2$, say with respect to a rectangle $\mathcal{P} = (-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}) \times (-\frac{\pi}{\beta}, \frac{\pi}{\beta})$, are decomposed into

$$\underline{u} = \operatorname{curl}\operatorname{curl} \varphi \underline{k} + \operatorname{curl} \psi \underline{k} + \underline{f}$$

with functions φ, ψ having the same periodicity as $\underline{u}$ and vanishing mean value over $\mathcal{P}$. The mean flow $\underline{f}$ depends on z only with first and second components f_1, f_2 and constant third component (cf. [2]). Let Φ_s be any sufficiently smooth

Received for publication October 1993.

†Partially supported by Grant-in-Aid for Scientific Research (No. 05640202), the Japan Ministry of Education.

AMS Subject Classifications: 76E99.