Differential and Integral Equations, Volume 7, Number 3, May 1994, pp. 863 – 872.

ON INSTABILITY OF EVOLVING HYPERSURFACES

YOSHIKAZU GIGA* AND KAZUYUKI YAMA-UCHI
Department of Mathematics, Hokkaido University, Sapporo 060, Japan

In the memory of Professor Peter Hess

Abstract. A general parabolic evolution equation is considered for a closed hypersurface in Euclidean space. All stationary solutions are shown to be Lyapunov unstable if the normal velocity of a hypersurface depends only on its normal and second fundamental form and is independent of its position. Instability of time periodic solution is also discussed.

1. Introduction. We consider the initial value problem of an evolution of a closed hypersurface Γ_t in $\mathbb{R}^n$,

$$V = f(\mathbf{n}, -\mathbf{A}). \tag{1}$$

Here $\mathbf{n}$ is an inward unit normal vector field on Γ_t and V is normal velocity in the direction of $\mathbf{n}$; $\mathbf{A} = -\,\mathrm{d}\,\mathbf{n}$ denotes the second fundamental form. We shall prove that all stationary solutions S of (1) is Lyapunov unstable provided that (1) is (nondegenerate) parabolic. This generalizes a recent work of Ei and Yanagida [2] where they assumed that f depends on $\mathbf{A}$ only through its mean curvature. Their method is completely different from ours. They linearized equations around stationary solutions and appealed to spectral analysis. Their method also applies to an equation depending on space variables but invariant under translation while ours does not apply to such an equation. We simply use a distance function of S and appeal to the strong maximum principle. We believe our proof is simpler than theirs for (1) with $\mathbf{A}$ replaced by mean curvature.

Our method also applies to instability of periodic solutions of

$$V = f(t, \mathbf{n}, -\mathbf{A}), \tag{2}$$

where f is time periodic. We show that periodic solutions S_t are unstable unless second fundamental form vanishes somewhere on S_t for all t. As an application, we show that the curvature of periodically evolving curves in the plane are unstable.

2. Parabolic evolution equations. We formulate our equations as in [4]. Let E be a bundle over the unit sphere S^{n-1} of the form

$$E = \{(\bar{p}, Q_{\bar{p}}(X)) \in S^{n-1} \times \mathbb{S}_n : X \in \mathbb{S}_n\}$$

Received for publication October 1993.
*Partially supported by the Inamori Foundation.
AMS Subject Classifications: 35B10, 35B35, 35K65.