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RESONANCE AND COULOMB FRICTION

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In Memoriam: Peter Hess (Zürich)

Abstract. Existence and uniqueness of ω-periodic solutions to

$$x'' + \mu \operatorname{sgn} x' + x = \varphi(t)$$

is studied by means of monotonicity and degree arguments for multivalued maps in case $\mu > 0$ and φ is continuous ω-periodic.

1. Introduction. We remember, in particular, how enthusiastic Peter Hess became about monotone operators more than 20 years ago; see e.g. [7] and his references given there. Today we use some monotonicity to get ω-periodic solutions of equation

$$x'' + \mu \operatorname{sgn} x' + x = \varphi(t) \tag{1}$$

which is considered as $x'' + x \in \varphi - \mu \operatorname{Sgn} x'$ almost everywhere with

$$\operatorname{Sgn} \gamma = \begin{cases} \operatorname{sgn} \gamma & \text{if } \gamma \neq 0 \\ [-1, 1] & \text{if } \gamma = 0; \end{cases}$$

in other words solvability means existence of a measurable selection $w(t) \in \operatorname{Sgn} x'(t)$ almost everywhere such that $x'' + \mu w(t) + x = \varphi(t)$ almost everywhere. Usually x'' will have only finitely many jumps, and x may have deadzones, say $x(t) \equiv c$ in $[\sigma, \tau]$, in which case $w(t) = (\varphi(t) - c)/\mu$ on this interval. Since this paper is a kind of continuation of [4], [5], let us first mention some.

2. Preliminaries. For the non-resonant case ($\omega \neq 2k\pi$ for all $k \geq 1$), Theorem 1 in [5] says that equation (1) has an ω-periodic solution, for every $\mu \geq 0$. There we had special φ, which was not important, but also an unduly long proof by means of a homotopy argument which is redundant. Notice that the corresponding first order system is $y' \in Ay + F(t, y)$ almost everywhere, with

$$F(t, y) = \{0\} \times (\varphi(t) - \mu \operatorname{Sgn} y_2) \quad \text{and } A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

and the ω-periodic solution of $y' = Ay + f(t)$ with ω-periodic f is given by

$$y(t) = \int_0^{\omega} \Gamma(t, s) f(s)\, ds,$$

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