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THE DAMPED WAVE EQUATION IN A NON-CYLINDRICAL DOMAIN

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Dedicated to the memory of Peter Hess

1. Introduction. In this paper we shall derive existence, uniqueness and regularity results for the solution u of the following equation with damping $\rho > 0$

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = \Delta(u + \rho \frac{\partial u}{\partial t}) & \text{in } Q, \\ u + \rho \frac{\partial u}{\partial t} = 0 & \text{on } \Gamma_t,\ 0 < t < T, \\ u(0, x) = u_0(x),\ \ \frac{\partial u}{\partial t}(0, x) = u_1(x) & \text{in } \Omega_0, \end{cases} \tag{1.1}$$

where $Q = \bigcup_{0<t<T}\{t\} \times \Omega_t$ and $\Gamma_t = \partial\Omega_t$. Physical motivations for studying problem (1.1) came from continuum mechanics, see [1] for details.

Here we assume that Ω_t is a bounded open set of $\mathbb{R}^N$ with a C^2 boundary which depends smoothly on t in a suitable sense to be discussed below.

This work is the continuation of a previous one by P. Cannarsa, G. Da Prato and J.P. Zolésio, [1]. Only the regularity results are new. Exactly as in [1] we view (1.1) as a non-autonomous evolution problem

$$\begin{cases} u''(t) = -A(t)(u(t) + \rho u'(t)), & 0 < t < T \\ u(0) = u_0, \ \ u,(0) = u_1, \end{cases} \tag{1.2}$$

in the fixed Hilbert space $H = L^2(\mathbb{R}^N)$.

Here, for each t, $A(t)$ is an unbounded operator with domain $D(A(t))$ which does depend on t. Actually, $A(t)$ is the unbounded operator corresponding to the continuous symmetric bilinear form

$$a_t(\varphi, \psi) = \int_{\mathbb{R}^N} \nabla\varphi \cdot \nabla\psi \, dx$$

defined for φ and ψ in V_t, where

$$V_t = \{\varphi \in H^1(\mathbb{R}^N) : \varphi = 0 \ \text{ on } \Gamma_t\}.$$

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