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EXISTENCE OF SOLUTIONS FOR SOME NON-LINEAR COOPERATIVE SYSTEMS

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Dedicated to the memory of Peter Hess

Abstract. In this paper we study the following cooperative elliptic systems and we show that the necessary and sufficient condition for having the Maximum Principle implies the existence of a solution for any $f_i \in L^{p'}$ for the system:

$$\begin{cases} -\Delta_p u_i = \sum_{j=1}^{j=n} a_{ij} \mid u_j \mid^{p-2} u_j + f_i \text{ in } \Omega \\ u_i = 0 \text{ on } \partial\Omega, \end{cases} \tag{S}$$

where Ω is a smooth and bounded domain in $\mathbb{R}^N$, Δ_p the p-Laplacian is defined by $\Delta_p u := \operatorname{div}(\mid \nabla u \mid^{p-2} \nabla u)$, $p > 1$, and where the coefficients a_{ij} $(1 \le i, j \le n)$ are constant and $a_{ij} \ge 0$ for $i \ne j$ (cooperative systems).

When $p = 2$, we also obtain a Fredholm alternative for existence of solutions.

1. Introduction. In this paper we study the following system:

$$\begin{cases} -\Delta_p u_i = \sum_{j=1}^{j=n} a_{ij} \mid u_j \mid^{p-2} u_j + f_i \text{ in } \Omega \\ u_i = 0 \text{ on } \partial\Omega, \end{cases} \tag{S}$$

where Ω is a smooth and bounded domain in $\mathbb{R}^N$, Δ_p the p-Laplacian is defined by $\Delta_p u := \operatorname{div}(\mid \nabla u \mid^{p-2} \nabla u)$, $p > 1$, the coefficients a_{ij} $(1 \le i, j \le n)$ are constant and $a_{ij} \ge 0$, for $i \ne j$ (cooperative systems), and $f_i \in L^{p'}(\Omega)$, with p' defined by $\frac{1}{p} + \frac{1}{p'} = 1$.

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