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BOUNDED H_∞-CALCULUS FOR ELLIPTIC OPERATORS

HERBERT AMANN AND MATTHIAS HIEBER
Math. Institut, Universität Zürich, Winterthurerstr. 190, CH-8057 Zürich, Switzerland

GIERI SIMONETT
Department of Mathematics, UCLA, Los Angeles, California 90024

In memoriam Peter Hess

Abstract. It is shown, in particular, that L_p-realizations of general elliptic systems on $\mathbb{R}^n$ or on compact manifolds without boundaries possess bounded imaginary powers, provided rather mild regularity conditions are satisfied. In addition, there are given some new perturbation theorems for operators possessing a bounded H_∞-calculus.

0. Introduction. It is the main purpose of this paper to prove — under mild regularity assumptions — that L_p-realizations of elliptic differential operators acting on vector valued functions over $\mathbb{R}^n$ or on sections of vector bundles over compact manifolds without boundaries possess bounded imaginary powers. In fact, we shall prove a more general result guaranteeing that, given any elliptic operator $\mathcal{A}$ with a sufficiently large zero order term such that the spectrum of its principal symbol is contained in a sector of the form $S_{\theta_0} := \{ z \in \mathbb{C} ; \ |\arg z| \leq \theta_0 \} \cup \{0\}$ for some $\theta_0 \in [0, \pi)$, and given any bounded holomorphic function $f : \overset{\circ}{S}_\theta \to \mathbb{C}$ for some $\theta \in (\theta_0, \pi)$, we can define a bounded linear operator $f(\mathcal{A})$ on L_p, and an estimate of the form

$$\| f(\mathcal{A}) \|_{\mathcal{L}(L_p)} \leq c \, \| f \|_\infty$$

is valid. This means that elliptic operators possess a bounded H_∞-calculus in the sense of McIntosh [16]. Choosing, in particular, $f(z) := z^{it}$ for $t \in \mathbb{R}$, it follows that $\mathcal{A}$ possesses bounded imaginary powers (cf. Section 2 below for more precise statements).

There are two main reasons for our interest in this problem. First, it is known (cf. [22], [24]) that the complex interpolation spaces $\big[E, D(A)\big]_\theta$ coincide with the domains of the fractional powers A^θ for $0 < \theta < 1$, provided A is a densely defined linear operator on the Banach space E possessing bounded imaginary powers. Second, by a result of Dore and Venni [10], the fact that A possesses bounded imaginary powers is intimately connected with 'maximal regularity results' for abstract evolution equations of the form $\dot{u} + Au = f(t)$. Both these results are of great use in the

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