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BOUNDARY STABILIZATION FOR THE STRETCHED STRING EQUATION

MARIUS TUCSNAK
Université d'Orléans, Département de Mathématiques, B.P. 6759, 45067 Orléans, France
and
Institute of Mathematics of the Romanian Academy

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Abstract. We consider a quasilinear partial integro-differential equation modelling the vibrations of a nonlinear string. In the presence of an appropriate boundary damping, we prove a global existence and uniqueness result, for "small" initial data. By the use of suitably chosen Lyapunov functionals, we also show the exponential decay of the energy for any strong solution and we give precise estimates of the rate of decay.

1. Introduction. In recent years, important progress has been made in the feedback boundary stabilization for distributed parameter systems of importance in structural dynamics, including some nonlinear models. However, with the exception of [5], only semilinear models were considered. This case is by now well understood (cf. [7] and [8]).

The aim of this paper is to study the boundary stabilization of a quasilinear partial integro-differential equation modelling the vibrations of a nonlinear string. More precisely, we shall consider the following initial and boundary value problem:

$$\frac{\partial^2 u}{\partial t^2}(x,t) - \Big\{a + b\int_0^1 \big[\frac{\partial u}{\partial x}(y,t)\big]^2 dy\Big\}\frac{\partial^2 u}{\partial x^2}(x,t) = 0, \tag{1.1}$$

$0 < x < 1$, $t \geq 0$, with $a > 0$, $b \geq 0$,

$$u(0,t) = 0,\ \frac{\partial u}{\partial x}(1,t) = -\alpha\frac{\partial u}{\partial t}(1,t),\ t > 0, \tag{1.2}$$

where α is a positive constant,

$$u(x,0) = u^0(x), \quad \frac{\partial u}{\partial t}(x,0) = u^1(x), \tag{1.3}$$

where u represents the transverse deflection of the string. Equation (1.1) is called the stretched string equation and a lot of papers have been devoted to the initial and boundary value problem formed by (1.1), (1.3) and the homogeneous Dirichlet boundary conditions

$$u(0,t) = u(1,t) = 0. \tag{1.4}$$

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