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THE BELLMAN EQUATION FOR CONSTRAINED DETERMINISTIC OPTIMAL CONTROL PROBLEMS

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Abstract. We consider a class of Bellman equations arising in time-optimal control problems with constraints which depend on the state of the dynamical system. The value function is characterized as the unique viscosity solution of a suitable Dirichlet problem in free boundary form, provided it is continuous. A sufficient condition for the continuity of the value function is given. The PDE theory is applied to give a "verification theorem" for the control problem.

0. Introduction. In this paper we consider a class of optimal control problems for dynamical systems described by differential inclusions, by using the method of Dynamic Programming. We study the problem of minimization of a given performance criterion over all pairs trajectory-control which solve the differential inclusion:

$$\begin{cases} y_x'(t) \in f(y_x(t), U(y_x(t))) \\ y_x(0) = x, \end{cases} \tag{0.1}$$

where $U(\cdot)$ is a set valued map which expresses constraints on the control.

We may find problems of this form in various disciplines; for example, Economics, Mechanics and Physics: see [2], [3], [9], [16].

The problem that we consider is a special case of a control problem with constraints expressed in implicit form by the condition $(y,u) \in \mathcal{U}$, where $\mathcal{U} \subset \mathbb{R}^N \times \mathbb{R}^M$ is a given set. Special cases of this constraint are the deterministic control problem usually studied in the literature, where the control is selected in a given compact set $U \subset \mathbb{R}^M$ (and so $\mathcal{U} = \mathbb{R}^N \times U$), and the problem with constraints on the state ($y(t)$ has to remain in a given closed set $\overline{\Omega}$ for all time), and so $\mathcal{U} = \overline{\Omega} \times U$, U as before (see work of Soner [21]). Control problems for dynamical systems with evolution governed by differential inclusion are studied by several authors: see Cesari [9], Clarke [10], Aubin and Frankowska [3], Zaremba [23], the recent survey of Aubin [1]. Frankowska [15] studied the Bellman equation for finite horizon problems in Bolza form by using the "contingent epiderivatives", while here we study constrained control problems with payoff involving the first exit time from a given set, in the framework of the recent theory of viscosity solution for Bellman equations (see [19], [12], [11], [18]). After this work was completed we learned of the paper of Mirica [20] dealing with related problems.

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