Differential and Integral Equations, Volume 6, Number 5, September 1993, pp. 1041–1043.

A NOTE ON PALAIS-SMALE CONDITION IN THE SENSE OF SZULKIN

D. GOELEVEN
Département de Mathématique, Facultés Universitaires ND de la Paix
B–5000 Namur, Belgium

(Submitted by: Jean Mawhin)

1. Introduction. It has been observed ([1]) that, for a Gâteaux differentiable lower semicontinuous function bounded from below on a Banach space, the Palais-Smale condition implies coercivity. Recently, critical point theory has been generalized by Szulkin ([4]) to the nonsmooth case, and for this, he introduced a compactness condition of Palais-Smale type.

Our aim in this Note is to generalize Caklovic-Li-Willem's result [1] to the nonsmooth case using the Palais-Smale condition as given by Szulkin.

2. Preliminaries. Let X be a real Banach space and let J be a function on X satisfying the following hypothesis:

(H) $J = \Phi + \gamma$, where $\Phi : X \to \mathbb{R}$ is a Gâteaux differentiable lower semicontinuous function and $\gamma : X \to (-\infty, +\infty]$ is a convex proper lower-semicontinuous function.

A point $u \in X$ is said to be a critical point of J if it satisfies the following variational inequality:

$$\langle \Phi'(u), v - u\rangle + \gamma(v) - \gamma(u) \geq 0, \quad \text{for each} \quad v \in X,$$

or equivalently,

$$0 \in \Phi'(u) + \partial\gamma(u).$$

Szulkin has proved ([4]) that Mountain Pass theorem of Ambrosetti, theorems of Clark and Ambrosetti and of Rabinowitz concerning the existence of multiple critical points for even functions, and some related results remain valid for such a function J as defined above with $\Phi \in C^1(X, \mathbb{R})$. For these functions, Szulkin introduced the following compactness condition:

(P.S.) If $\{u_n\}$ is a sequence such that $J(u_n) \to c \in \mathbb{R}$ and

$$\langle \Phi'(u_n), v - u_n\rangle + \gamma(v) - \gamma(u_n) \geq -\varepsilon_n \cdot \|v - u_n\| \text{ for each } v \in X,$$

where $\varepsilon_n \to 0$, then $\{u_n\}$ possesses a convergent subsequence.

Received for publication May 1992.
AMS Subject Classification: 58F05, 49A27.