ABSTRACT DIFFERENTIAL EQUATIONS
AND NONLINEAR DISPERSIVE SYSTEMS

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1. Introduction. Let M be a closed operator $D(M) \subset X \to X$, where X is a Banach space. We shall assume that M is invertible with a bounded inverse M^{-1}. Let Y be another Banach space such that

$$D(M) \subset Y \subset X \quad \text{with continuous injections.} \tag{1.1}$$

Let f be an operator $f : [0, \infty) \times Y \to X$, where $f(t, u)$ is continuous in $t \geq 0$ and locally Lipschitz continuous in the variable u, with the Lipschitz constant uniform in t for $t \in [0, T]$, any $T < \infty$. In this paper we shall study questions related to local existence, a priori bounds (global existence) and regularity of the solutions to the following Cauchy problem

$$\begin{cases} Mu_t(t) = f(t, u(t)), & t \geq 0, \\ u(0) = u_0 \in D(M). \end{cases} \tag{1.2}$$

There are several equations of physical interest which can be recast in the form (1.2). For example, the Benjamin-Bona-Mahony equation studied in [1] and its extensions considered in various papers [2, 8]. We quote in a particular way the very recent paper [4] because in fact our abstract results give many of their assertions when applied to that dispersive equation.

By providing a theory of solvability and regularity for the abstract model (1.2), we will be able to recover and to extend some of the recent results obtained in the literature for some specific partial differential equation models. Notice that our examples treat a specific differential operator M, but that many other operators, like the ones considered in [8], can be considered as well. The outline of the paper is as follows.