Differential and Integral Equations, Volume 6, Number 6, November 1993, pp. 1507–1517.

EXACT MULTIPLICITY RESULTS FOR TWO CLASSES OF BOUNDARY VALUE PROBLEMS

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(Submitted by: A.R. Aftabizadeh)

Abstract. We give a complete description of the set of positive solutions for two classes of boundary value problems, involving both convex and non-convex nonlinearities.

1. Introduction. We consider Dirichlet problem of the type

$$u'' + \lambda f(x,u) = 0 \quad \text{on } (a,b), \quad u(a) = u(b) = 0 \tag{1.1}$$

for two classes of nonlinearities depending on a parameter λ, and prove existence and exact multiplicity results for positive solutions. Our main tool is a bifurcation theorem of M.G. Crandall and P.H. Rabinowitz [3]. In both cases we obtain a complete description of the set of positive solutions for all values of λ.

We begin by considering convex in u nonlinearities. We assume that $f(x,u)$ is even in x relative to the midpoint $\frac{a+b}{2}$ of the interval (a,b). This allows us to get rather detailed information about the solution $u(x)$ of (1.1), as well as on the solution of the corresponding linearized problem, which makes it possible to apply the Crandall-Rabinowitz bifurcation theorem. We outline our arguments next. The problem is easily solvable for small λ. Since we assume $f(x,u)$ to be positive on $[-1,1]$ and grow superlinearly in u, there are no positive solutions for large λ. We show that the curve of solutions "bends back" at a critical λ_0, and then study the curve after the bend, obtaining rather detailed description of the solution curve. We show then that there are no other positive solutions by excluding all other possible situations.

Our second class involves non-convex nonlinearities of the type studied by T. Ouyang [7]. Using the results of those papers, we can apply a similar analysis to obtain a complete description of the set of positive solutions.

Our approach appears to be quite general. We have some multiplicity results for polynomial in u nonlinearities, which we defer to future publications. Our results can be used to describe the structure of global attractors for the corresponding parabolic problems, see e.g., J. Hale [5].

Next, we list some background results. Recall that a function $\varphi(x) \in C^2(a,b) \cap C^0[a,b]$ is called a supersolution of (1.1) if

$$\varphi'' + \lambda f(x,\varphi) \le 0 \quad \text{on } (a,b), \quad \varphi(a) \ge 0, \ \varphi(b) \ge 0. \tag{1.2}$$

A subsolution $\psi(x)$ is defined by reversing the inequalities in (1.2). The following result is standard.

Received April 1992.
AMS Subject Classification: 34B15.