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MIXTURES, CONVECTION, DIFFUSION, AND ADSORPTION UNDER THE ASSUMPTION OF EQUILIBRIUM PARTITIONING

SETH F. OPPENHEIMER†

Department of Mathematics, Mississippi State University, Mississippi State, MS 39762

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Abstract. A model is developed for the behavior of a mixture of contaminants flowing through an absorbing medium in an inert carrier. We assume that the concentrations of absorbed contaminant and unabsorbed contaminant are in equilibrium at all times. Three special cases are then analyzed.

1. Introduction. This work arises from an attempt to model the desorption of a contaminant from a saturated salt water sediment. Here we develop our most general model for the adsorption–desorption behavior of a mixture of m contaminants from a saturated porous absorbent with diffusion and prescribed convection. Our goal is to make this model more generally known and to analyze several special cases. A similar problem with a single absorbable component and no diffusion has been studied in [21], [15], [16], and [17]. Experimental results may be found in [14].

We will assume that there are m possibly absorbable contaminants flowing through an n-dimensional porous absorbent in an inert carrier, perhaps water. We shall view the absorbent as filling a C^1-domain G in $\mathbb{R}^n$. We assume that at each time t and each x in G the velocity field for the carrier $v(x, t)$ is divergence free; this amounts to incompressibility. The function $u_i(x, t)$ will represent the concentration of contaminant i in the carrier unbound to the absorbent at point x and time t. Similarly, $a_i(x, t)$ represents the concentration of contaminant i absorbed at point x and time t. We will write $u(x, t) = (u_1(x, t), \ldots, u_m(x, t))$ and $a(x, t) = (a_1(x, t), \ldots, a_m(x, t))$. Finally, $K_i(x, u(x))$ will represent the diffusion coefficient for contaminant i.

The change in the total mass of contaminant i in a smooth subdomain Ω of G from time t to time $t + \Delta t$ is given by

$$\int_\Omega u_i(x, t + \Delta t) + a_i(x, t + \Delta t) - u_i(x, t) - a_i(x, t)dx.$$

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