Differential and Integral Equations, Volume 6, Number 6, November 1993, pp. 1299–1312.

A FREE BOUNDARY PROBLEM FOR A QUASILINEAR ELLIPTIC EQUATION PART I: RECTIFIABILITY OF FREE BOUNDARY

SEIRO OMATA†
Department of Mathematics, Kitami Institute of Technology

(Submitted by: Yoshikazu Giga)

Abstract. In this paper, a free boundary problem for a quasilinear elliptic equation derived from a variational problem is treated. This type of problem has been studied by Alt, Caffarelli and Friedman. The existence and interior Hölder continuity of a minimizer are shown. Moreover, if a domain is two dimensional, one can show Lipschitz continuity of a minimizer under some growth condition for coefficients. It immediately follows that the free boundary consists of a countably rectifiable set.

1. Introduction. In [1] and [2], Alt, Caffarelli and Friedman have treated free boundary problem for a minimizer of the following type of functionals for

$$I(u) = \int_\Omega \left(F(|\nabla u|^2) + Q^2(x)\chi(\{u>0\})\right) d\mathcal{L}^n, \quad \text{for } u : \Omega(\subset \mathbb{R}^n) \to \mathbb{R},$$

where $Q(x)$ is a given measurable function with $0 < Q_{\min} \le Q(x) \le Q_{\max}$, and χ denotes a characteristic function and $\Omega(\subset \mathbb{R}^n)$ is a domain (may be unbounded) with Lipschitz boundary. Here and in the sequel we denote $\{x \in \Omega : \ u(x) > 0\} = \{u > 0\}$. The case $F(t) = t$ was treated in [1], and the case $F(t)$ belonging to $C^{2,1}[0,\infty)$, with $F(0) = 0$, $0 < c \le \frac{\partial F}{\partial t} \le C$ and $0 \le (1+t)\frac{\partial^2 F}{\partial t^2} \le C$, in [2]. It has been proved that if $Q(x)$ is Hölder continuous, then the free boundary $\partial\{u > 0\}$ are $C^{1,\beta}$-curves in any compact subset $\tilde{\Omega}$ contained in Ω. These results are applied to solve jet and cavitational flow problems (see [3], [4], [5] and [11]).

We here consider another physical model of this problem especially for two dimensional case. In other words, this problem describes a physical model of "peeling off a thin rubber which adhered to a plane". The procedure of this physical experiment is as follows. First, we expand a thin rubber film equally so that the rubber may have the same tension ($\equiv T$) in any place and any direction and stick it on a coordinate plane. We regard the portion of the plane where rubber is adhered as domain(Ω). Next, we peel the rubber off vertically from the plane by seizing the edge of it. The graph of a function $u : \Omega \to \mathbb{R}$ describes the shape of the rubber. Now we measure energy of the peeling off procedure under the following two assumptions.

Received November 1992.
†Partly supported by the Grants-in-Aid for Scientific Research, The Ministry of Education, Science and Culture, Japan.
AMS Subject Classifications: 49J20, 35R35.