Differential and Integral Equations, Volume 7, Number 1, January 1994, pp. 293–299.

EXISTENCE OF SIGNED SOLUTIONS FOR A SEMILINEAR ELLIPTIC BOUNDARY VALUE PROBLEM

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(Submitted by: A.R. Aftabizadeh)

Abstract. In this note we consider the problem of existence of signed solutions for the semi–linear elliptic boundary value problem

$$\triangle u + u_+^q - u_-^p = 0 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where Ω is an open bounded subset of $\mathbb{R}^N$ with smooth boundary, $0 < q < 1$ and $2 < p+1 < 2^*$. Using well known results we see that this problem has a positive and a negative solution. We show that for certain large domains there exists a solution that changes sign. The proof of our result is based on the Mountain Pass Theorem.

1. Introduction. In this note we consider the problem of existence of signed solutions for the semi–linear elliptic boundary value problem

$$\left.\begin{aligned} \triangle u + u_+^q - u_-^p = 0 \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega \end{aligned}\right\} \tag{1.1}$$

where Ω is an open bounded subset of $\mathbb{R}^N$ with smooth boundary, $0 < q < 1$ and $2 < p+1 < 2^*$. Throughout this note we denote by 2^* the critical Sobolev's exponent, that is $2^* = \frac{2N}{N-2}$ if $N \geq 3$ and $2^* = +\infty$ if $N = 2$. Also $u_+ = \max\{u, 0\}$ and $u_- = \max\{-u, 0\}$.

In order to motivate our problem let us consider the one–dimensional version of (1.1),

$$u'' + u_+^q - u_-^p = 0 \text{ in } [0, L], \quad u(0) = u(L) = 0. \tag{1.2}$$

Problem (1.2) can be solved by direct integration. Indeed, by considering the initial value problem

$$u'' + u^r = 0, \quad u(0) = 0,\ u'(0) = a > 0$$

Received May 1992.
†Partially supported by D.I.U.C and FONDECYT under grant 91-0813.
‡Partially supported by FONDECYT under grant 91-1212.
AMS Subject Classifications: 35J65, 58E99.