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ON THE TIME MAP OF A NONLINEAR TWO POINT BOUNDARY VALUE PROBLEM

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Abstract. We study the bifurcation of the time map of positive solutions of the nonlinear two-point boundary value problem $u'' + f(u) = 0$, $-L < x < L$, $u(-L) = u(L) = 0$ for $f(u) = -(u-a)(u-b)(u-c)$ satisfying $0 \le a < b < c$ and $c > 2b - a$. Under one additional hypothesis on the cubic polynomial f, we are able to show that the time map not only has exactly one critical point, a minimum, but is also a strictly convex function by modifying a time map technique introduced by J. Smoller and A. Wasserman [1]. Combined with some results of J. Smoller and A. Wasserman [1] or of S.-H. Wang [2], our result implies that for some cubic polynomials $f(u) = -(u-a)(u-b)(u-c)$ with fixed numbers $0 < a < b$, the time map has exactly one critical point, a minimum, for any number $c > 2b - a$. Our method can be generalized to general functions f with $f''' < 0$.

1. Introduction. In this paper we study the bifurcation of the time map of positive solutions of the nonlinear two-point boundary value problem

$$u'' + f(u) = 0, \ -L < x < L, \tag{1.1}$$

$$u(-L) = u(L) = 0, \tag{1.2}$$

where $2L > 0$, the interval length, is a real bifurcation parameter, and $f(u)$ is a cubic polynomial of the form

$$f(u) = -(u-a)(u-b)(u-c) \tag{1.3}$$

satisfying

$$0 \le a < b < c \tag{1.4}$$

and

$$c > 2b - a; \tag{1.5}$$

that is, we assume that the cubic polynomial f has three distinct nonnegative real roots and $\int_a^c f(u)\,du > 0$. Thus there exists a unique γ with $b < \gamma < c$ satisfying

$$\int_a^\gamma f(u)\,du = 0. \tag{1.6}$$

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