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PERIODIC SOLUTIONS OF CERTAIN PLANAR RATIONAL ORDINARY DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

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Abstract. Existence of periodic solutions is proved for systems of two differential equations of the form $\dot{x} = f(t,x,y)$, $\dot{y} = g(t,x,y)$, where f and g are continuous functions T-periodic in t which are rational functions of $(x,y) \in \mathbb{R}^2$ with 0 as the only singularity, for each t, and satisfying some further assumptions. The proof is based on the study of Poincaré map in $\mathbb{R}^2 \setminus \{0\}$ and on a modification of the Conley's concept of isolating block. Explicit examples are provided.

1. Introduction. We study first order semilinear differential equations whose right-hand sides are rational functions of $(x,y) \in \mathbb{R}^2$ with the origin as the only singularity and with time-dependent T-periodic coefficients. By using the complex-number notation, one may express any such equation in the form

$$\dot{z} = \sum_{k,l \in \mathbb{Z}} b_{k,l}(t) z^k \overline{z}^l, \tag{1.1}$$

where the sum is finite, it may be extended over negative as well as positive integers, and $b_{k,l}$ are T-periodic continuous complex-valued functions. We may call such equations of Fourier-Laurent type, since for any function $f(t,x,y)$, T-periodic in t, its finite expansion, Fourier in t and Laurent in (x,y), takes the form of the right-hand side of (1.1). We will assume that the lowest order terms (i.e., those corresponding to the minimum value of $k+l$) satisfy a so-called ellipticity condition and the highest order terms satisfy a so-called hyperbolicity condition.

Analogous equations of Fourier-Taylor type, i.e., when the right-hand side is a polynomial in z and $\overline{z}$, were studied in [18]. Let us note that if the sum in (1.1) extends over negative values of k and l only, then a change of variable $w = z^{-1}$ brings (1.1) to an equation of Fourier-Taylor type and results of [18] apply. We will therefore consider the cases where both positive and negative degree terms appear.

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