Differential and Integral Equations, Volume 7, Number 2, March 1994, pp. 583–592.

STURMIAN COMPARISON METHOD IN OSCILLATION STUDY FOR DISCRETE DIFFERENCE EQUATIONS, II

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1. Introduction. This paper is a continuation of [1] in which an efficient method has been suggested by us for studying the oscillatory properties of discrete difference equations and inequalities of the delay, advanced, and mixed type. This method is based on obtaining Sturmian-type comparison theorems and on their applications. The main advantage of the suggested method is the possibility of estimating the intervals on which the solutions didn't change their sign.

The estimation of the intervals could not be achieved by using alternative approaches. Moreover, the method allows obtaining new results even in such a well-known standard problem as describing the conditions achieving the oscillation of all solutions of the equation. In particular, [1] contains the solving of one from the open problems by Ladas (Problem 8.6, [2]).

The subject of the present paper is the investigation of the difference inequality

$$l[x]_i \equiv x(i+1) - a(i)x(i) + \sum_{k=1}^{M} b^{(k)}(i)x(i-k) + \sum_{l=1}^{N} c^{(l)}(i)x(i+l) \leq 0, \tag{1}$$

and the corresponding equation

$$l[x]_i = 0, \quad i \in \mathbb{N}. \tag{2}$$

The following difference inequality is intimately connected with (1):

$$\tilde{l}[y]_i \equiv y(i-1) - \tilde{a}(i)y(i) + \sum_{k=1}^{M} \tilde{b}(i+k)y(i+k) + \sum_{l=1}^{N} \tilde{c}^{(l)}(i-l)x(i-l) \geq 0. \tag{3}$$

Here we use the following notation:

$$\mathbf{N} = \{1, 2, \dots\}; \ \langle n, m\rangle = \{n, n+1, \dots, m\} \subset \mathbf{N};$$
$$\sum_{n=p}^{p-i-1} a(n) \overset{\text{def}}{\equiv} -\sum_{n=p-i}^{p-1} a(n); \quad \sum_{n=p}^{p-1} a(n) \overset{\text{def}}{\equiv} 0.$$

Let us formulate the Sturmian-type comparison theorem (see [1]) which serves as the basis for our investigation.

Received May 1992.
*Research supported by Israel ministry of absorption and ministry of science.
AMS Subject Classification: 39A10.