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THE STOKES PROBLEM AND VECTOR POTENTIAL OPERATOR IN THREE-DIMENSIONAL EXTERIOR DOMAINS: AN APPROACH IN WEIGHTED SOBOLEV SPACES

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Abstract. This paper solves a number of problems related to the Stokes system in exterior domains of $\mathbb{R}^3$. The equations are set in weighted Sobolev spaces, for any finite integer weight, that describe the growth or decay of the functions at infinity. The results established include "inf-sup" conditions for the divergence, characterization of the vector potential of divergence-free vector fields, existence, uniqueness and regularity of the solution of the Stokes problem.

1. Introduction and preliminaries. This paper is concerned with some aspects of flows past obstacles. More precisely, the obstacle Ω' is a bounded domain of $\mathbb{R}^3$ (possibly empty) with a boundary Γ that is at least Lipschitz-continuous and Ω denotes the complement of $\overline{\Omega'}$, in other words, the exterior of Ω'. This paper solves the Stokes problem and constructs the vector potentials of divergence-free vector fields in Ω. As auxiliary results, it gives some insight on the gradient and divergence operators and it yields a number of very useful theorems on equivalent norms. It follows a previous work of the author [18] in which all the above-mentioned properties are established in the simplified case where Ω is the entire space $\mathbb{R}^3$.

The data are assumed to belong to a weighted Sobolev space, and the solution of the Stokes problem or the vector potential are sought for in some (other) appropriate weighted Sobolev space. These spaces describe very conveniently the growth or decay of the functions at infinity. They were introduced and studied by Hanouzet in [26] and a wide range of basic elliptic problems were solved in these spaces by Giroire in [22]. The presentation of this paper is fairly simple because it is based on many results proved in [22] and [18].

The originality of this paper is that the theorems that we prove are valid for a wide range of weights at infinity, thus taking sharply into account the growth or decay of the functions. Some particular cases are well-known results that were already derived by several authors. But the earlier results are only set in the basic space W_0^1, although the name of this space is not always explicitly mentioned, as they were published either before or independently of Hanouzet [26]. And the latter results, sharper and more sophisticated, are set in L^p spaces, which is a different

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