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RESOLVENT OPERATORS AND WEAK SOLUTIONS OF INTEGRODIFFERENTIAL EQUATIONS

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Abstract. Equations from heat conduction and viscoelasticity are written as

$$x'(t) = A\Big[x(t) + \int_0^t F(t-s)x(s)ds\Big] + f(t),\ t \geq 0,\ \ x(0) = x_0, \tag{1}$$

in a Banach space X, where A is a closed and densely defined operator and $F(t)$ is a bounded operator for $t \geq 0$. We obtain the equivalence between the resolvent operator of equation (1) and solutions of

$$\begin{aligned} \frac{d}{dt}\langle x(t), v\rangle &= \langle x(t) + \textstyle\int_0^t F(t-s)x(s)ds, A^*v\rangle + \langle f(t), v\rangle,\ t \geq 0, \\ x(0) &= x_0, \end{aligned} \tag{2}$$

where A^* is the adjoint of $A, v \in D(A^*)$, and $\langle\, ,\, \rangle$ denotes the pairing between X and its dual X^*. The result also enables us to unify many concepts about solutions of equation (1).

1. Introduction.

Consider the equation

$$x'(t) = Ax(t) + \int_0^t B(t-s)x(s)ds + f(t), \quad t \geq 0,\ \ x(0) = x_0, \tag{1.1}$$

in a Banach space X with A and $B(\cdot)$ closed and densely defined operators. An important topic in the study of equation (1.1) is the *resolvent operator*, which is defined to be a bounded operator-valued function $R(\cdot)$ satisfying

$$\begin{aligned} \frac{d}{dt}R(t)y &= AR(t)y + \int_0^t B(t-s)R(s)y\,ds \\ &= R(t)Ay + \int_0^t R(t-s)B(s)y\,ds,\ \ t \geq 0,\ \ y \in D(A), \end{aligned} \tag{1.2}$$

where D is the domain. One reason why the resolvent operator is important is that if equation (1.1) has a resolvent operator $R(\cdot)$, then (e.g., see [6]) classical solutions of equation (1.1) are given by

$$x(t) = R(t)x_0 + \int_0^t R(t-s)f(s)ds,\ \ t \geq 0, \tag{1.3}$$

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