Differential and Integral Equations, Volume 7, Number 2, March 1994, pp. 501–507.

ON PERIODIC SOLUTIONS OF GENERALIZED LIÉNARD EQUATIONS*

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(Submitted by: Kenneth L. Cooke)

Abstract. In this paper a new sufficient condition is obtained for the generalized Liénard equation

$$\ddot{x} + f(x, \dot{x})\varphi(\dot{x}) + g(x) = 0$$

to have at least one nonzero periodic solution. This result extends and improves several known results in the literature.

1. Introduction. Since 1942, the following nonlinear oscillating equation

$$\ddot{x} + f(x, \dot{x})\dot{x} + g(x) = 0 \tag{1}$$

or its equivalent differential system

$$\dot{x} = v, \quad \dot{v} = -f(x, v)v - g(x) \tag{2}$$

has been extensively investigated by many authors, for example see, Levinson and Smith [7], Dragilev [4], Francesca [1], Cesari [3], Graef [5], Haro [6], Sansone and Conti [10], Ponzo and Wax [8], Wu [13], Zheng [14,15] and the references cited therein. Equation (1) is a generalization of the well known Liénard equation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0 \tag{3}$$

whose qualitative behavior has been widely studied in the literature concerning nonlinear oscillations, e.g., see [3] and [10].

Our aim in this paper is to consider a more general equation than (1), namely

$$\ddot{x} + f(x, \dot{x})\varphi(\dot{x}) + g(x) = 0 \tag{4}$$

or its equivalent system

$$\dot{x} = v, \quad \dot{v} = -f(x, v)\varphi(v) - g(x) \tag{5}$$

and to give sufficient conditions on f, g and φ to guarantee that it has at least one nonzero periodic solution. The method used in this paper is to construct an annular

Received in revised form March 1993.
*This work was supported by the NSF of China.
AMS Subject Classification: 34C05.