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ON A STATIONARY PROBLEM FOR THE COMPRESSIBLE NAVIER-STOKES EQUATIONS: THE SELF-GRAVITATING EQUILIBRIUM SOLUTIONS

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1. Introduction. In this paper we study the stationary motion of a compressible, viscous and heat-conductive fluid in a bounded domain Ω of $\mathbb{R}^3$, in the presence of self-gravitation, with the velocity field satisfying a slip boundary condition instead of the usual adherence condition. A part of the paper is also devoted to the study of self-gravitating equilibrium solutions. The stationary Navier-Stokes equations are

$$\begin{cases} -\mu\Delta u - \nu\nabla \operatorname{div} u + \nabla P(\rho,\Theta) = \rho[f - (u\cdot\nabla)u - \nabla U], \\ \operatorname{div}(\rho u) = 0, \\ -\chi\Delta\Theta + c_v\rho u\cdot\nabla\Theta + \Theta p'_\Theta \operatorname{div} u = \rho g + \alpha(u) \quad \text{in } \Omega, \end{cases} \tag{1.1}$$

where $u(x) = (u_1(x), u_2(x), u_3(x))$ denotes the velocity field, $\rho(x)$ the density, $\Theta(x)$ the absolute temperature. Here the pressure $P = P(\rho,\Theta)$ is a known smooth function of ρ and Θ; U is the Newtonian gravitational potential given by

$$U(x) = -G\int_\Omega \frac{\rho(y)}{|x-y|}dy, \tag{1.2}$$

where G is the constant of gravitation; μ and ν represent the viscosity coefficients, χ is the coefficient of heat conductivity and c_v is the specific heat at constant volume. In order to avoid technicalities we will assume that the coefficients μ,ν,χ,c_v are constant. We will also assume the following physical constraints: $\mu > 0$, $\nu > \frac{\mu}{3}$, $\chi > 0$, $c_v > 0$. Finally, f denotes the given external force field, g the given heat supply and $\alpha = \alpha(u)$ the dissipation function

$$\alpha(u) = 2\mu T(u) : T(u) + (\nu-\mu)(\operatorname{div} u)^2$$

where $T(u) = \frac{1}{2}(D_iu_j + D_ju_i)_{1\le i,j\le 3}$ is the deformation tensor,

$$T(u) : T(v) = \frac{1}{4}\sum_{i,j=1}^{3}(D_iu_j + D_ju_i)(D_iv_j + D_jv_i), D_i = \frac{\partial}{\partial x_i}.$$

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