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EXISTENCE AND UNIQUENESS OF COEXISTENCE STATES FOR A PREDATOR-PREY MODEL WITH DIFFUSION

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Abstract. We study a predator-prey model with a saturating interaction term, due to Holling and Tanner. We present numerical evidence for the existence of multiple coexistence states for a range of parameter values, including the possibility of two *stable* coexistence states. Rigorous proofs of the existence of regions of parameter space which have two coexistence states (one stable and one unstable) are given. Exact results on regions in parameter space which have a unique stable coexistence state are also described. Some simple criteria which can be used to analyze the number of possible coexistence states are also given.

1. Introduction.

Consider the model

$$\begin{aligned} -\Delta u &= \lambda u - u^2 - b\frac{uv}{\gamma+u} \\ -\Delta v &= \mu v + c\frac{uv}{\gamma+u} - v^2 \end{aligned} \qquad \text{in } \Omega, \tag{1.1}$$

$$u = v = 0 \quad \text{on } \partial\Omega,$$

where Ω is a bounded domain in $\mathbf{R}^N$ with a smooth boundary $\partial\Omega$, Δ stands for the Laplacian operator in $\mathbf{R}^N$, and λ, μ, $b \geq 0$, $c \geq 0$, $\gamma > 0$, are real numbers.

The problem (1.1) provides us with a simple prototype, modeling the interactions between a predator, with population density $v(x)$, and a prey, with population density $u(x)$, inhabiting the region Ω. Both species in this model are assumed to grow logistically in absence of the other. The self-interaction terms between both species we consider here are often known as Holling-Tanner interactions. These interaction terms take into account the saturation of the predator in presence of a high prey population. The constant γ measures the saturation level of the predator, [9, 12]. During the last years the analysis of this and other related models has generated

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